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2023

(IMMC)

Summary Sheet

An Integrated Land Planning Model Based on Overall Profits and Geographical Features

Summary

In recent years, as the urbanization goes on, businesses need to devise appropriate land use plans in order to both maximize their profits and to fit into the geographical factors of a specific area. Thus, with an attempt to develop an algorithm that can choose what to build in an area considering both the profits and geological attributes, we proceed as follows.

To determine the advantage of facilities, we use **Analytical Hierarchy Process** to establish a **Metric System** for 8 facilities at first. We have taken several important factors like income into account. These results give the investors details about the overall benefits of building the facilities.

We then develop an **Area Division Model** to differentiate the areas suitable for building different facilities. In order to make our calculations more accurate, we use a **Differential Division** to divide the areas into small grids and to calculate the appropriateness of building facilities in a specific grid. In order to quantify this appropriateness, we devise different weights that indicate the geological needs of each facility and draw a **Heat Map** of the appropriateness for building for each facility.

After getting these heat maps, we select all of the grids that are more than 70 **percent** appropriate for building a specific facility and develop a **Clustering Algorithm** to compile the scattered grids into one. We also use the hierarchy to decide the facility to build in the specific area with several appropriate facilities to maximize the profits that the investors will receive. However, we still take the geological factors into consideration because some areas are suitable for building several facilities.

For Task 3, we test the ability of our model in adjusting to the change in population and need for entertainment due to the construction of a big factory. We have also added several facilities in order to adjust to these needs. Our model tells us that living houses, outdoor complex and other things that are associated with the social and entertainment needs of people will be more appealing to investors. Thus, more areas will be built in these facilities. Besides, we have developed a **Separating Algorithm** because some facilities may have similar geological requirements as well as social benefits, such as living houses and outdoor sports complex. We will first consider these two facilities as a whole facility in our area division model. Then we will use our separating model to determine the location of the facility in order to maximize the profits.

Finally, we have tested our model in other areas by changing the geological conditions. For example, we have analyzed our model judgement of what facility to build in an area that is under extensive drought.

Keywords:Analytical Hierarchy Process, Area Division, Clustering Division, Separating algorithm

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1 Introduction

1.1 Background

Land resource is a non-renewable resource, which provides the basis for the living of humans. As the process of industrialization and urbanization goes on, land resources are becoming more and more precious and essential now. Therefore, land use planning will play an important role in the development of a certain area. The quality of a land use plan will affect the results of the construction a lot and will determine whether the land can be used fully or not. An outstanding plan will result in a greater income for business planners and a balance between environment and construction, while a bad plan will only waste a large amount of money and can't ensure that business planner can cover their cost.

Thus, to come up with a great plan, people needs to take a wide range of factors into considerations. Both the human factors and natural factors will influence the suitability of land for each facility. However, as more factors are influencing our results, simply analysis by people will be more inaccurate. They cannot consider all the data as a whole and will fail to notice some critical factors. Therefore, a model that can successfully process data and help people to make a plan is in urgent need.

To solve this problem, our team take a part of land in the rural area in Syracuse, NY, USA as an example to develop a math model, using their terrain statistics and other relevant data from the Internet, so as to try to establish a math model that can draw up a excellent construction plan through the careful considerations of complex data.

Figure 1 Views in Syracuse, NY, USA

1.2 Problem Restatement

In this essay, the following questions will be coped with:

1. A facility will have pros and cons in different aspects. Therefore, we should develop a metric

that define the meaning of ”best use” so that we can compare different facilities directly and make a choice after considering all their characteristics,

2. Then we will choose at least two of the options mentioned in the prompt and analyze their value through the our metric. Besides, we will also justify the value that our metric focus on, so that our metric will be more reasonable and convincing.

3. As there will be many incidents that will influence the results when people are applying this model in real life, our model should also be flexible to new variables. By introducing a new variable, which is about a new semiconductor fabrication facility that is built nearby, into our previous model, we will reevaluate our metric, so as to show the practicality of our model.

4. At last, we will extend our model to other places with different local environments and improve the generalizability of our model, illustrating that the same way can be used when the conditions vary.

2 Assumptions and Notations

2.1 General Assumptions

- **Assumption1:** People’s satisfaction has nothing to do with the income that the facility provides to the community

  Justification: In order to take people’s satisfaction as a factor of our metric, it is vital to assume that this factor will not be affected by other factors.
- **Assumption2:** The fertility of the soil depends mainly on the distance of the soil to the water

  Justification: Since the proximity to rivers will make the soil moist, it is reasonable to assume that this kind of soil will be more conducive to the growth of plant. Besides, rivers will carry some mineral containing nitrogen or phosphorus. These chemicals will promote the growth of plants’ leaves and roots.
- **Assumption3:** The sunshine rate of a certain district is directly attributed to the plant-cover of this area.

  Justification: According to [1], we know that crop growth and yield are strongly affected by sunlight. As a result, we assume that the plant-cover of a certain region is directly influenced by the sunshine rate of a certain region.
- **Assumption4:** Lower elevation is more beneficial to farms, either crop or grazing.

  Justification: Lower elevation provides more oxygen to stocks and plants, which is beneficial to their growth.
- **Assumption5:** Most facilities cannot achieve full function in a small area, this means that the areas of a certain facilities often have an area minimum. **Justification:** Most facilities have an area minimum because facilities built in a small area will cause unusually high construction cost. In addition, facilities concerning large structures like cross-country skiing cannot be built in small areas.
- **Assumption6:** The judging matrix A does not change with the change of adding the factories. **Justification:** The building of the factory mainly changes the judging matrix of C_i, which is B_i. The relative importance between factors will change a little because the changes will be mainly reflected on the facilities. So we assume that the judging matrix A will not change.

2.2 Notations

Table 1 Variables and Meanings

Symbols	Description
P_1	Outdoor Sports Complex
P_2	Cross-country Skiing Facility
P_3	Crop Farm
P_4	Grazing Farm
P_5	Regenerative Farm
P_6	Solar Array
P_7	Agrivoltaic Farm
P_8	Agritourist Centre
P_9	Living Quarter
P_{10}	Golf Course
A	Factor importance matrix
B_i	The judging matrix of factor C_i
B_i'	The judging matrix of factor C_i after the factory is built
E_A	The eigenvector of matrix A
E_i	The eigenvector of matrix B_i
E_i'	The eigenvector of matrix B_i'
CR_A	The coincidence indicator of matrix A
CR_i	The coincidence indicator of matrix B_i
G_1	Elevation
G_2	Plant Cover
G_3	Electric Signal
G_4	Fertility
G_5	Altitude Drop
F	Optimal center of the outdoor sports complex
A_i	The vertices of the convex polygon of the total living area
ρ	The living density in the area
S	Total area of the living quarter
S'	Total area of the outdoor sports complex
a_k	The complex number representing vertex A_k in the complex plane
w_{ij}	The weight of P_i to geological factors G_j
g_{ij}	The original data of the geological factor G_j of the ith small grid
K_{ij}	Value of appropriateness of the ith grid to the building P_j
m	Optimal values of g_{ij} for middle data

3 Task 1: Evaluation of Overall Profits

We use Analytic Hierarchy Process(AHP) to establish a metric [3]. In the process, we take six key aspects into consideration, which are income per square meter, relative benefits for the environment, people's satisfaction, construction cost, maintenance cost and local energy contribution.

In Figure 2, this is the process of AHP.

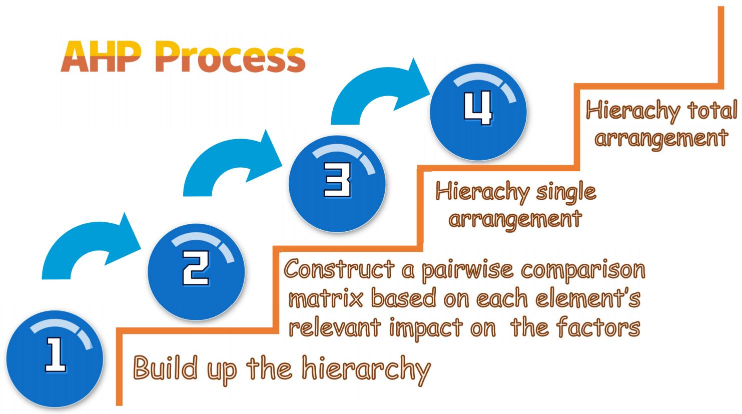


Figure 2 Four main steps of AHP

Table 2 Key Aspects in AHP Considerations

Symbol	Description
C_1	Income Per Square Meter
C_2	Relative benefits for the Environment
C_3	People's Satisfaction
C_4	Construction Cost
C_5	Maintenance Cost
C_6	Local Energy Contribution

We then construct seven judgment matrix based on our assumption of these eight facilities as well as the importance of these factors. We use relative number to evaluate the importance of each factor. And here is a table representing the relative meaning of the numbers.

Table 3 Number Meanings

Symbol	Description
1	Same importance
3	relative importance
5	obvious importance
7	much importance
9	strong importance
$2, 4, 6, 8$	the mid value between the values above

Besides, in the process of Analytic Hierarchy Process, we only do comparisons between two choices, instead of considering every options together. Therefore, our logic of comparison needs a certain degree of consistency and the CR value in AHP will help us to identify the consistency of our matrix.

This is the matrix representing the importance of all the six factors. In this factor importance matrix A , the element on ith row and j th column represents the relative importance of factor C_i to C_j.

$$A = \begin{bmatrix} 1 & 6 & 3/2 & 4 & 3 & 8 \\ 1/6 & 1 & 1/4 & 2/3 & 1/2 & 1 \\ 2/3 & 4 & 1 & 3 & 2 & 5 \\ 1/4 & 3/2 & 1/3 & 1 & 3/4 & 2 \\ 1/3 & 2 & 1/2 & 4/3 & 1 & 8/3 \\ 1/8 & 1 & 1/5 & 1/2 & 3/8 & 1 \end{bmatrix} \quad E_A = \begin{bmatrix} 0.39 \\ 0.06 \\ 0.26 \\ 0.09 \\ 0.13 \\ 0.05 \end{bmatrix} \tag{1}$$

$$CR_A \approx 0.0017$$

3.1 AHP Matrices

The six judging matrices of the eight facilities will be provided in the Appendices.

After getting these matrices, we calculate eigenvectors of these matrices using Python. The eigenvectors and the code will also be listed in the Appendices.

Using these eigenvectors, we are able to calculate the scores of all facilities P_i:

$$\begin{bmatrix} E_1 & E_2 & E_3 & E_4 & E_5 & E_6 \end{bmatrix} \cdot E_A \tag{2}$$

where E_i is a partitioned matrix in the first matrix. Then the result vector (calculations are in the appendices) will be:

$$\begin{bmatrix} 0.2657 \\ 0.3196 \\ 0.1045 \\ 0.1045 \\ 0.149 \\ 0.2695 \\ 0.2995 \\ 0.2624 \end{bmatrix} \tag{3}$$

The value on the ith row in the resulting matrix represent the final overall value of P_i. We can see that P_2 have the highest overall score of 0.3196, this means that the most confident choice will be building cross-country skiing facilities. Building Agrivoltaic Farms (0.2995) and Solar Arrays (0.2695) are also reasonable options.

3.2 Brief Justification of the weight of the factors

Next, we will justify our former assumptions. For C_1 to C_6, we've hypothesized that $C_1 > C_3 > C_5 > C_4 > C_2 > C_6$, and it is beyond dispute that the net profit may be the most paramount factor when making choices, since the land use planning is partly determined by business planners. Besides, as the planning is also decided by community leaders, we should not neglect the importance of residents' satisfaction of the construction plan.

Additionally, if the cost of construction and maintenance is too high, the business planners will need more capital for the investment, which will create a barrier for the development plan. However, these factors may not be as essential as C_1 and C_3, because the profit and satisfaction will determine whether the plan can be put into effect or not.

Also, as environment protection is becoming more and more crucial these days, especially after the pandemic, by no means should we ignore the relative benefits of the construction for the environment. Moreover, though local energy contribution is also worth noticing, most of the facilities are not capable of producing energy, which means it will not be the best indicator to determine. Hence, $C_1 > C_3 > C_5 > C_4 > C_2 > C_6$ shall be an effective criterion.

Now, we will select several representative C_i to make precise analysis. When it comes to C_1(income per square meter), the most vital element we should concern is how much profit the construction can make. From [4], we know that Regenerative Farms can profit more than usual Crop Farms and Grazing Farms.

In addition, though Solar Arrays and Agrivoltaic Farms may need more initial funding, they are able to provide large quantities of consistent incomes for up to 50 years[3]. Besides, Sports Complex and Skiing Facility can get a high income, since they will also facilitate the development of service industry. However, skiing facility will not be as profitable as the outdoors sports complex because it depends on seasons.

As for C_2(relative benefits for the environment), to begin with, regenerative farms have the potential to change the landscape and improve the climate, which can do better than ordinary ones.[4]

What's more, Agrivoltaic Farms and Agritourist Centres combine modern technology with traditional agriculture, which is good to the environment since solar energy is a clean and green resource of energy. However, the modern constructions(P_1 and P_2,that are Sports Complex and Skiing Facilities) might result in habitat degradation and pollution[6].

4 Task 2: Land Planning Considering Geographical Factors

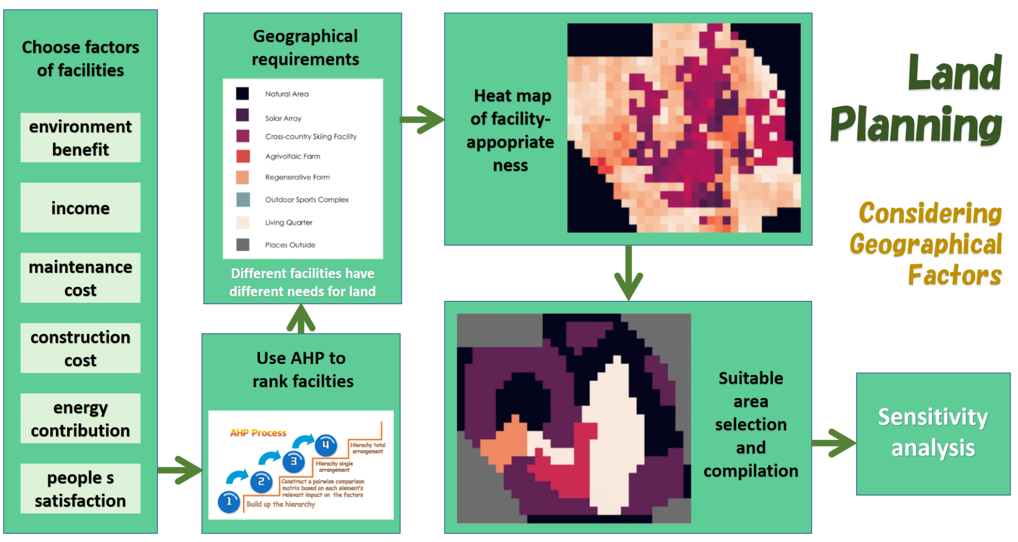


Figure 3 Flow chart of our model in Task 2

In Figure 3, this is flow chart of our model.

Since we can divide the land into several portions, the geographical factors of the given map needs to be taken into consideration when building a certain facility. We have taken the factors elevation, plant cover, electric signal, fertility and the altitude drop into account. We will call them factor G_1, G_2, G_3, G_4 and G_5.

4.1 Differential Division

In order to decide which facility to build in each area, we divided the whole district into small parts like this and measure these geographical data mentioned above. The district divided by little squares, with 25 roles and 30 columns, as shown in Figure 4 below:

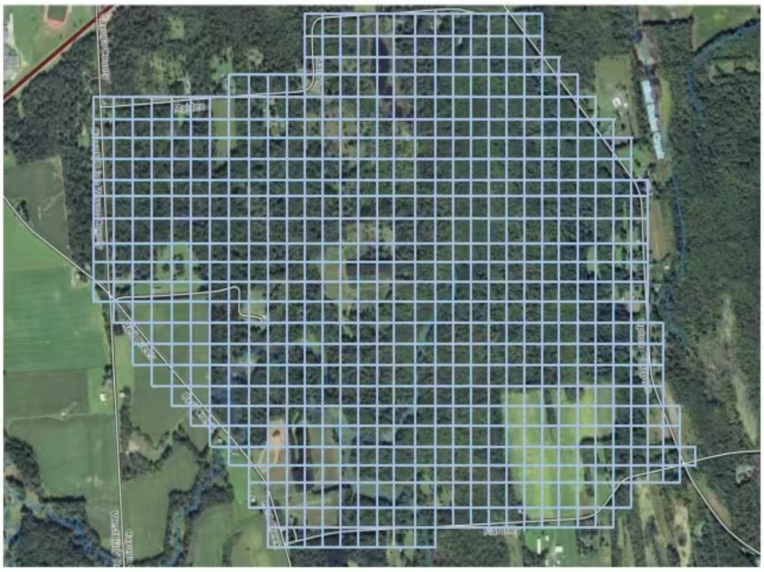


Figure 4 Grid Map of Syracuse, NY, USA

We then determine the most appropriate geographic characteristics for each facility. N/A means we do not consider this factor, Max means the characteristic should be as high as possible, Min means that the data is as small as possible, and Mid means that the data should be close to the right amount. For instance, for Altitude Drop (G_5), the requirement for Cross-country Skiing Facility (P_2) is Max, while the requirement for Crop Farm (P_3) is Min. It means that the Cross-country Skiing Facility needs a higher altitude drop and the Crop Farm needs a lower one. All of the optimal values are listed in the table below:

Table 4 Appropriate Geographic Characteristics

$Facility$	G_1	G_2	G_3	G_4	G_5
P_1	N/A	Min	Max	N/A	Min
P_2	Max	Min	Max	N/A	Max
P_3	Min	Min	N/A	Max	Min
P_4	Min	Mid	N/A	Max	Min
P_5	Min	Min	N/A	Max	Min
P_6	Max	Min	Max	N/A	Min
P_7	Mid	Mid	Max	Max	Min
P_8	Min	Min	Max	Max	Min

4.2 Normalization and Other Data Processing

After getting these optimal data from electronic maps, we then perform the normalization of our data. First, we will turn all our values into a value that is between 0 and 1 by dividing each data to the maximum of all the data.

Then, we will process the data set, so as to satisfy the requirement of Max, Min and Mid mentioned above.

If the magnitude of the value correlates positively with the appropriateness for a facility:

$$g'_{ij} = \frac{g_{ij}}{Max(g_{1j}, g_{2j}, ...g_{Nj})} \tag{4}$$

If the magnitude of the value correlates negatively with the appropriateness for a facility, we will directly use the data as a minimum data and we will use 1 to minus all of the data and turn into the form of maximum data. This means that:

$$g'_{ij} = 1 - \frac{g_{ij}}{Max(g_{1j}, g_{2j}, ...g_{Nj})} \tag{5}$$

For middle values, which mean that the optimal value lies between the maximum and the minimum value, we assume that the optimal value for g_{ij} for a particular j is m, then:

$$g'_{ij} = 1 - |m - g_{ij}| \tag{6}$$

Note that since the requirements of different facilities are not the same, it is possible that we need to do both the maximum normalization as well as the minimum normalization to a set of data.

4.3 Different weights

After processing the data, we will decide the weights of each of the factors to determine a value that demonstrates the appropriateness for a certain land to build a certain facility.

The weights are necessary. For example, Solar Arrays depend highly on the altitude drop as well as the plant cover in order to receive enough sunlight, so other two factors will be less important than these two. We will list all of the weights below:

Table 5 The weights of geographical factors

Facility	G_1	G_2	G_3	G_4	G_5
P_1	0	0.2	0.4	0	0.4
P_2	0	0.2	0.15	0	0.65
P_3	0	0.1	0.1	0.6	0.2
P_4	0	0.3	0.1	0.3	0.3
P_5	0	0.1	0	0.65	0.25
P_6	0.1	0.4	0.1	0	0.4
P_7	0.1	0.2	0.1	0.3	0.3
P_8	0	0.2	0.3	0.2	0.2

We regard the w_{ij} as the weight value in the ith row and jth column. This w_{ij} represents the weight of a facility P_i to geological factors G_j.

4.4 Heat map of facilities

Based on the weights and the normalized data. We can calculate a value of the appropriateness for a certain facility. We call this value of appropriateness: K_{ij}. It represents the appropriateness of the ith grid to building the facility P_j. We also assume that g_{ij} is the data of the geological factor G_j of the ith small grid.

Therefore, we will have:

$$K_{ij} = \sum_{k=1}^{6} g'_{ik} \cdot w_{jk} \tag{7}$$

In this way we can calculate the value of appropriateness for the ith grid K_{ij}.

The magnitude of K_{ij} correlates positively to appropriateness of this land to build the facility.

Then we will normalize the data of K_{ij} and get K'_{ij}, suppose N is the total number of grids we divide: Then we will have the normalized data:

$$K'_{ij} = \frac{K_{ij}}{Max(K_{1j}, K_{2j}, ...K_{nj})} \tag{8}$$

Then based on these values, we can draw a heat map showing the relative appropriateness of this facility. In the figures, the lighter the square is, the more suitable it is for a certain facility.

As we can see in the picture of Cross-Country Skiing Facility, only a part of the lower left corner satisfy the altitude drop for a Cross-Country Skiing Facility. While in the middle of the picture, most of the squares are covered in black, meaning that the place is too flat for the construction of skiing facility.

Besides, in the figure of Solar Arrays, it is evident that the center of the area has a dense forest. Therefore, they are not suitable for building Solar Arrays and it is reasonable that the center of the area is dark.

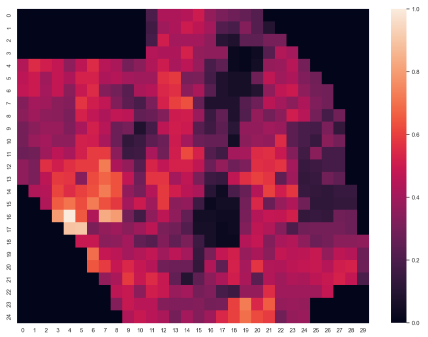


Figure 5 Heat Map of Skiing Facilities

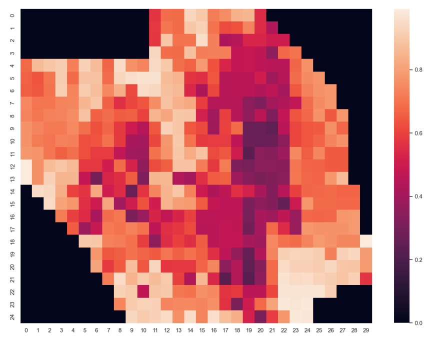


Figure 6 Heat Map of Solar Arrays

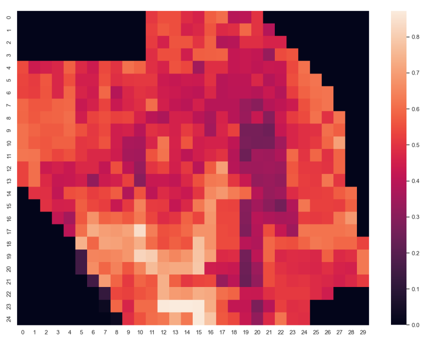


Figure 7 The Heat Map of Agrivoltaic Farms

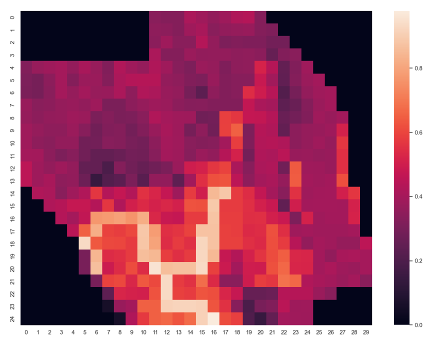


Figure 8 The Heat Map of Regenerative Farms

In Figure 5-8, these are the heat maps for the four facilities. According to these heat maps, we know that each small area's value ranges from 0 to 1. We will then use an algorithm to select the areas that are suitable for building a specific facility, selecting areas with value over a given constant.

4.5 Clustering Algorithm

A lot of the selected squares are scatteredly distributed, but facilities are often built in a centralized space. It will cost too much to build them sporadically. Therefore, we develop a clustering algorithm to centralize the blue squares, as is shown below:

If there are more than or equal to 5 blue squares around a white square, then we turn this square into blue. If there are less than or equal to 3 blue squares around a white square, then we will turn this square into white.

In Figure 9, this is the picture showing the exact process of our algorithm.

After repeating this process for several times, the scattered squares become more centralized, as in shown below:

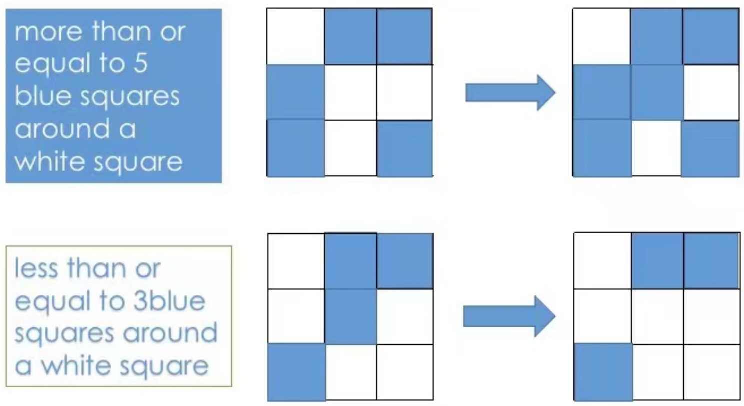
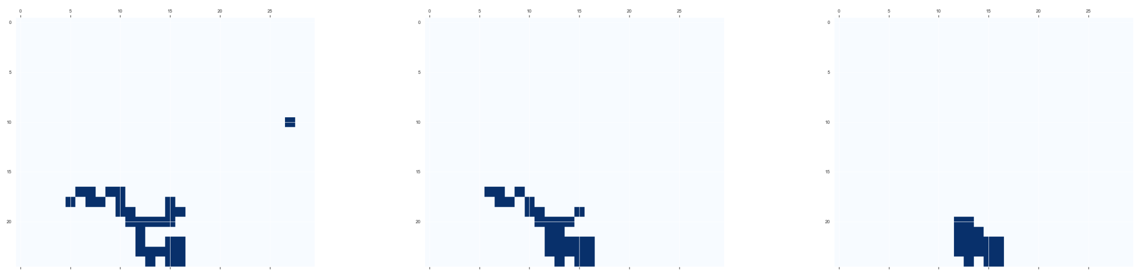


Figure 9 Clustering Algorithm

4.6 Block maps and Final results

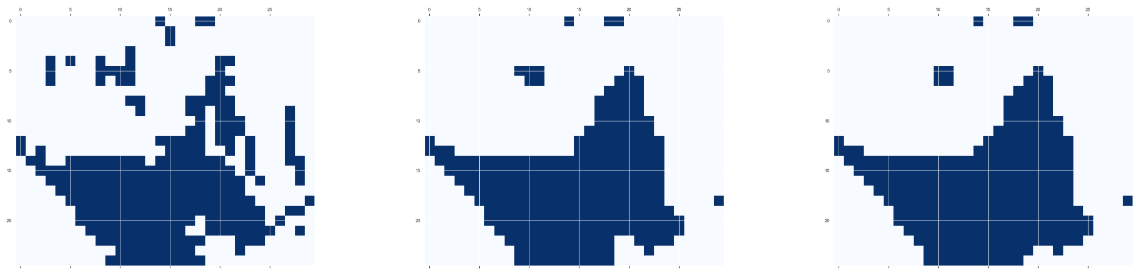
For Agrivoltaic Farm, we select areas whose values are more than or equal to 0.7 and repeat the process for two times:



(a) Initial (b) First Operation (c) Second Operation

Figure 10 Selected Areas of Agrivoltaic Farms

For Regenerative Farms, we select areas whose values are more than or equal to 0.6 and repeat the process for two times:



(a) Initial (b) First Operation (c) Second Operation

Figure 11 Selected Areas of Regenerative farm

In Figure 10 and 11, these are the transformation process of grid maps for Agrivoltaic Farm and Regenerative Farm.

We will then compile all of these 4 final pictures into a big picture. Note that there will be some areas that are suitable for several different facilities, we will choose the facilities of these areas by their values calculated by the AHP. The larger their value, the higher their priority.

Therefore, the hierarchy will be $P_2 > P_7 > P_6 > P_5$, which means that the importance rank is Cross-country Skiing Facility, Agrivoltaic Farm, Solar Array and Regenerative farm.

However, we see that some of the small areas are still separated from their large groups and people can't build a facility in those small areas. Since their appearance is inevitable, we have to carry out manual intervention in the last step, making every part of areas a big portion.

As is shown below, we turn two squares of regenerative farms in the middle into squares of Agrivoltaic Farm and two squares of Agrivoltaic Farms in the lower left part, next to the skiing facility, into squares of skiing facilities:

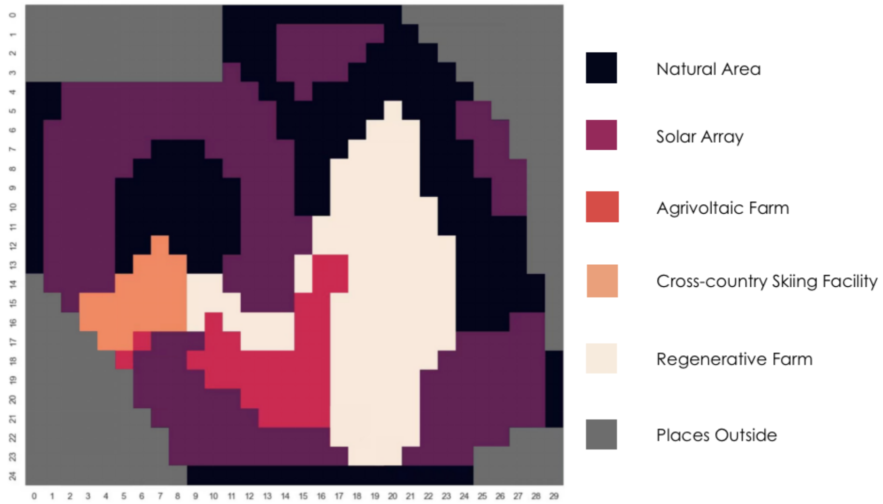


Figure 12 Original land use plan

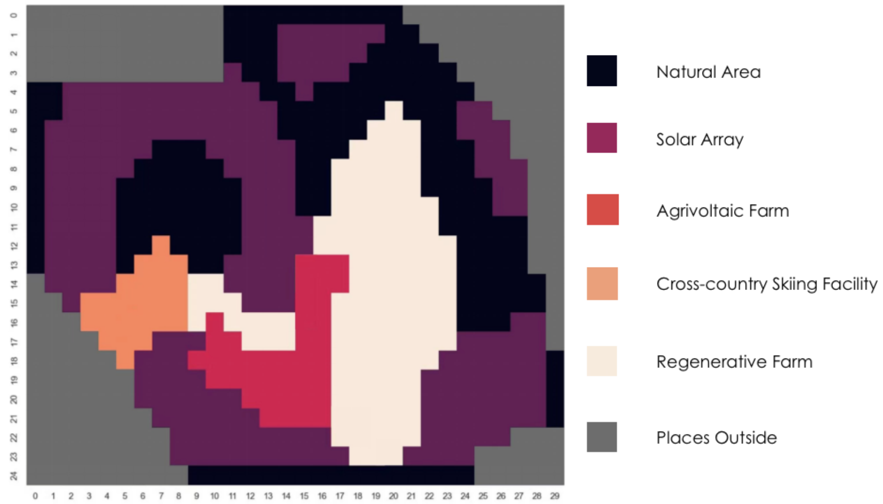


Figure 13 Land use plan after manual intervention

In figure 12 and 13, they show the final plan for Task 2 before and after manual intervention.

The purple areas are planned to build Solar Arrays, the white ones are planned to build Regenerative Farms. What's more, the orange areas are planned to build Cross-country Skiing Facilities and the red ones are planned to build Agrivoltaic Farm and the grey ones are areas out of the land use planning region.

Note that we have already covered most of the areas, we can still see that a small number of areas are black, not suitable for any facilities. It is either because their slopes are too high or because they have no wire connection. In this situation, building the facility is not economical, which means that the profit earned by facilities can't cover the cost of construction.

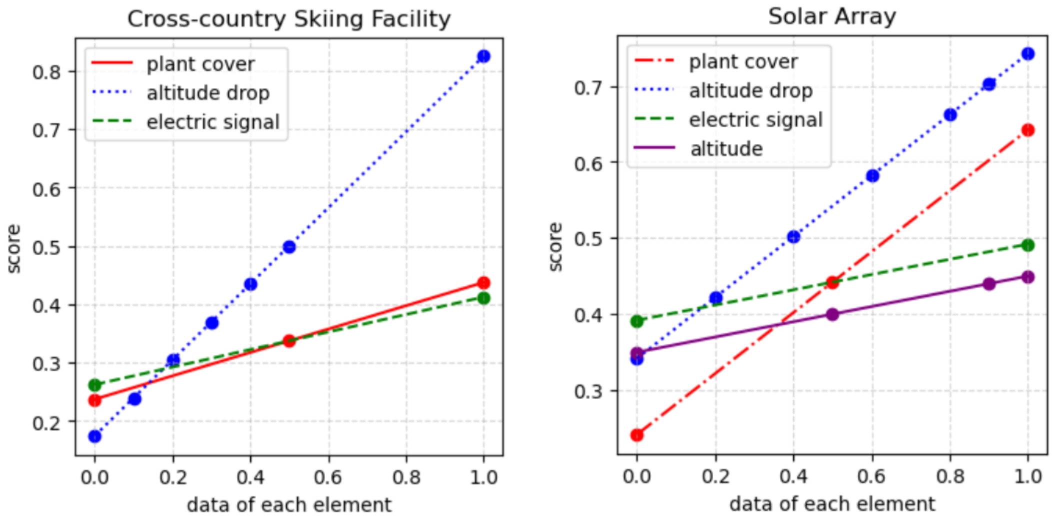
Therefore, there is no facility in these two areas, so as to preserve the environment and reduce cost.

4.7 Sensitivity Analysis

In order to test whether our results is sensitive to some factor, which means that when a certain factor changes a little, the results will change a lot, we need to do sensitivity analysis. We take the

Cross-Country Skiing Facility and Solar Array for example. The results are presented below:

Figure 14 Sensitivity Graph



In Figure 14, they are the sensitivity graph of different factors.

The larger the slope of the line is, the more sensitive the result is to the factor. Therefore, we can see that for Cross-Country Skiing Facility, the results are most sensitive to altitude drop. This is reasonable because it is common sense that skiing often requires high altitude drop. We can also discover that the results are less sensitive to plant cover and electric signal.

For Solar Arrays, the most sensitive two factors are altitude drop and plant cover, which is also reasonable because it is impossible to build solar panels in high altitude-drop. Also tree coverage will affect the sun energy the Solar Arrays received. Besides, the results are less sensitive to electric signals and altitude.

In summary, our model is effective generally for every data belong to interval [0,1], which means our model have high applicability.

5 Task 3: Model adjustment to changes in variables

In this task, the building of the large semiconductor fabrication facility will increase the population of this area, so there will be more people around this area and the need of entertainment as well as their living places need to be concerned. Therefore, with this new variable, some of the characteristic of our facility choice will change. We then rewrite some of the judging matrices.

We will add two more facilities in order to take this human factor into full consideration. We add a Living Quarter(P_9) and a Golf Course(P_{10}) and reevaluate them using AHP similar to task 1.

5.1 New AHP

We will not change the factor importance matrix in C_i, which is A. So we will only change the matrices of judging matrix of factor C_i. A brief justification and new AHP matrices are also provided in the appendices.

Similarly, we will add these to our final matrix and calculate the values of each facilities. The result vector is:

$$\begin{bmatrix} 0.1347 \\ 0.1203 \\ 0.0646 \\ 0.0646 \\ 0.0725 \\ 0.0986 \\ 0.0892 \\ 0.1061 \\ 0.1304 \\ 0.1055 \end{bmatrix} \tag{9}$$

We can see that this time, Outdoor Sports Complex(P_1) scores the highest(0.1347), Living Quarter scores in the second place(P_9), reaching 0.1304. And cross-country skiing facility scores the third place (P_2), reaching 0.1203.

5.2 New Weights

Table 6 Factors of metric

The optimal value of geographical factors for P_9 and P_{10}					
Facility	G_1	G_2	G_3	G_4	G_5
P_9	N/A	Mid	Max	N/A	Min
P_{10}	N/A	Min	Max	Max	Min

Table 7 Factors of metric

The weights of geographical factors for P_9 and P_{10}					
Facility	G_1	G_2	G_3	G_4	G_5
P_9	0	0.1	0.5	0	0.4
P_{10}	0	0.1	0.2	0.1	0.6

5.3 Additional Heat maps and results

We discover that the the requirements of P_9(Living Quarters)and P_1(Outdoor Sports Complex) are similar. If we still use the same way in task 2 to determine the areas of these two, these areas will largely overlap with each other. Therefore, we choose to first consider them together in the algorithm determining the areas. Then, we will invent a independent algorithm separating these two, instead of using their AHP value to determine which facility to build. The reason is that combining Living Quarter and Outdoor Sports Complex will create a better effect than either building Living Quarter or building Outdoor Sports Complex alone, as the combination can provide workers with a more pleasant community. In the algorithm below, we will call the total area P_{19}.

Since we have already drawn some heat graphs in the previous task, we will then draw the additional graphs of P_{19} (the combination) and P_{10}, the golf course. Here are the heat maps for these two:

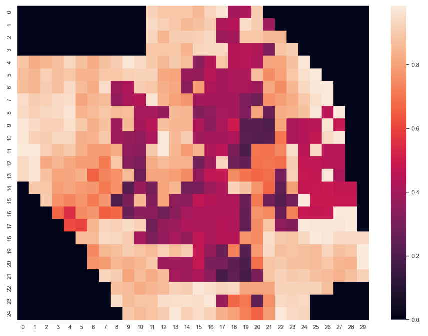


Figure 15 Heat Map of living area and outdoor sports center

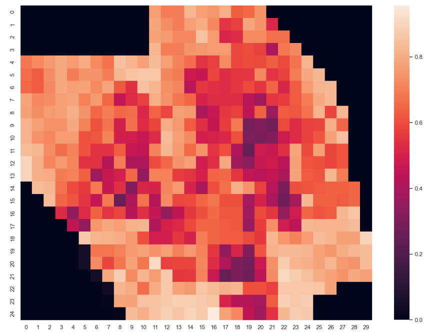
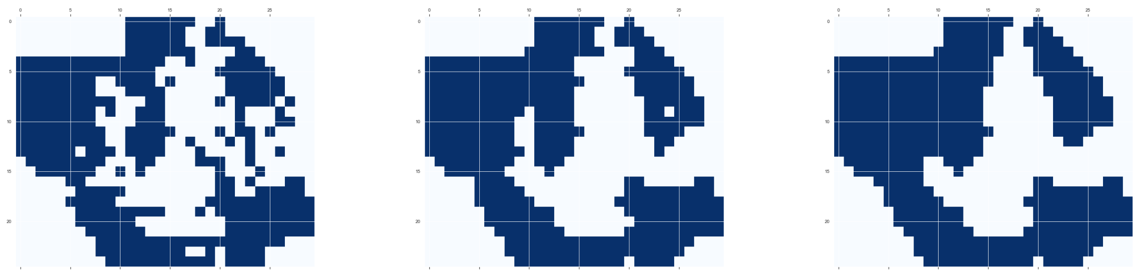


Figure 16 Heat Map of Golf Course

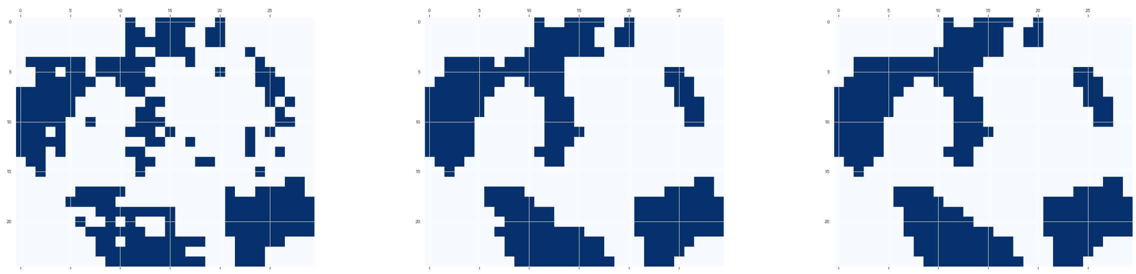
In Figure 15 and 16, they are the heat maps for living quarter and golf course.

For living quarter, we will first select all the areas whose value is greater than or equal to 0.7. For Golf Course, we will first select all the areas whose value is greater than or equal to 0.7. After that we will apply the algorithm to centralize the blue square for two times:



(a) Initial (b) First Operation (c) Second Operation

Figure 17 Selected areas of living quarter



(a) Initial (b) First Operation (c) Second Operation

Figure 18 Selected areas of Golf Course

In Figure 17 and 18, these are the transformation process of grid maps for Living Quarter and Golf Course.

We then will still use the algorithm to determine the areas. According to the value of AHP that we have calculated, we will consider facility P_{19}, P_2, P_{10},P_5,P_6, P_7. Then the hierarchy will be $P_{19} > P_2 > P_{10} > P_6 > P_7 > P_5$.

This means that the combination of the living quarters and the sports center is the most important, following Cross-country Skiing, Golf Course, Agrivoltaic Farm, Solar Arrays and Regenerative Farm.

And the resulting graph will be:

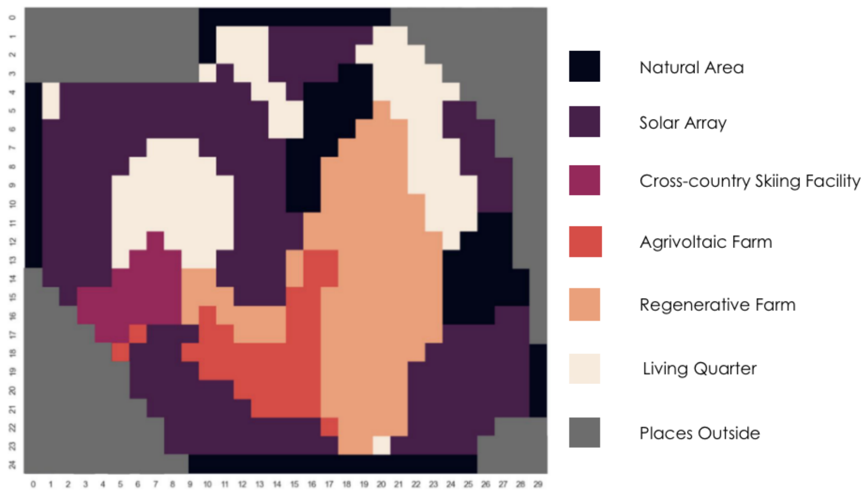


Figure 19 Original land use plan

In Figure 19, it is the plan for Task 3 before manual intervention.

Note that there is no part for golf course, as the construction cost of it is high and need to pay a lot for maintenance. Therefore, the AHP value for it is relatively low. In addition, its requirements are highly similar to other facilities that provide more profits, such as living quarter. Therefore, it leads to the consequence that there is no squares for golf courses. This result is consistent with people's judgments, meaning that our model are able to make a reasonable and logical land plan.

We will then correct some areas like we have done in the previous task and do the separation of Outdoor Sports Complex and Living Houses. We will get the final graph:

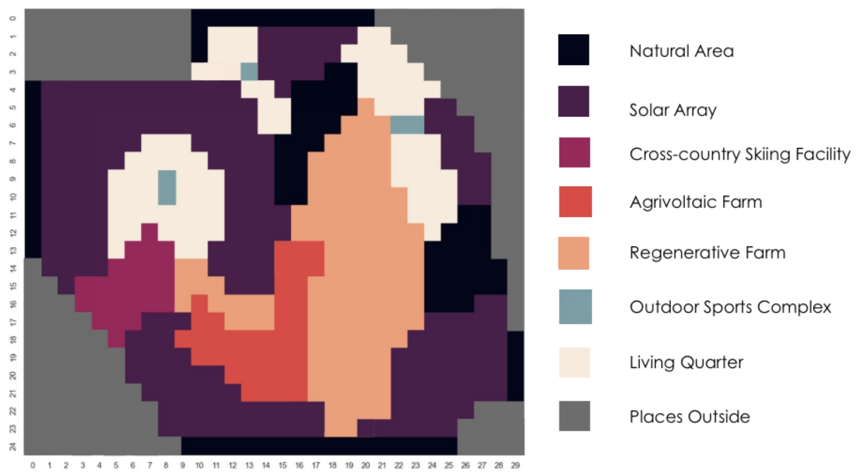


Figure 20 Land use plan after manual intervention

In Figure 20, it is the plan for Task 3 after manual intervention.

5.4 Separating Algorithm

We need to develop an algorithm separating the outdoor sports complex. Suppose that we will only build one sports complex for residents in a chunk of living quarter. The primary goal of building the facility is to minimize the total distance from all of the residents to the Outdoor Sports Complex. However, we do not know the actual houses in this living quarters. Therefore, we only need to find a point K that minimize the sum of the distance to the vertex because residents living on these vertexes are probably living far from the sports complex.

Suppose that the vertices of this convex polygon is A_i, $i = 1, 2...n$. Our aim is to minimize the value of:

$$\sum_{i=1}^{n} |FA_i| \tag{10}$$

The point F satisfying this function will have a characteristic: $\angle A_i F A_{i+1} = \frac{2\pi}{n}$, where $A_{n+1} = A_1$(The proof is given in the appendices). And we can use this function to find F.

After we have determined the center of the sports complex, we also need to find out the actual area of it. Suppose that the actual density of people living in a the whole area is ρ, the size of this sports complex is directly associated with ρ. Therefore, the area number of people living in this area is $\rho \cdot S$, where S is the total area of the living quarters, we assume that ρ is a constant value, then the outdoors sports complex area S'.

$$S' = k \cdot \rho \cdot S \tag{11}$$

where k is a constant number.

To determine the ratio of Living Quarters and Outdoor Sports Complex, we search the data about construction standard of residential community and decide the value of k and ρ.

In the picture above, there are three chunks of residential areas, having space of 0.12, 0.22 and 0.27 km^2 each.

Therefore, the sports center in the upper residential area will only take up one square and the sports center in the left and upper right living quarters will take up two squares. So we have added the Outdoor sports complex in the final areas in the picture above.

6 Task 4:Model influenced by regional development

We assume that the landform is the same as the one in previous tasks and the distinctions between different places are just influenced by regional development. Because of page limit, we only provide two situations here: a place is well-developed and a place is backward.

When it comes to the well-developed region, we need to notice that labour-intensive businesses will not be profitable since there won't be many residents in this area. As for entertaining facilities especially Golf Courses or Skiing facilities, they will be off great profit because people have time to enjoy their life cozily, and they will be easier to be built because people have high-end technology.

As for under-developed region, it need to be pointed out that farms and other kinds of labour-intensive industries will be very conductive due to high population density, while facilities requiring fewer people but vaster lands cannot make best use of lands and constructing them will not be the best choices.

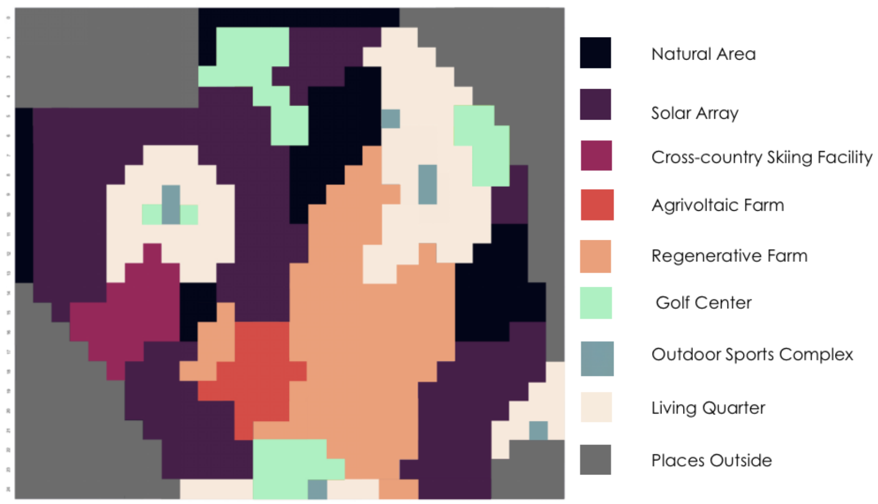


Figure 21 The Land Use Plan for Well-developed Region

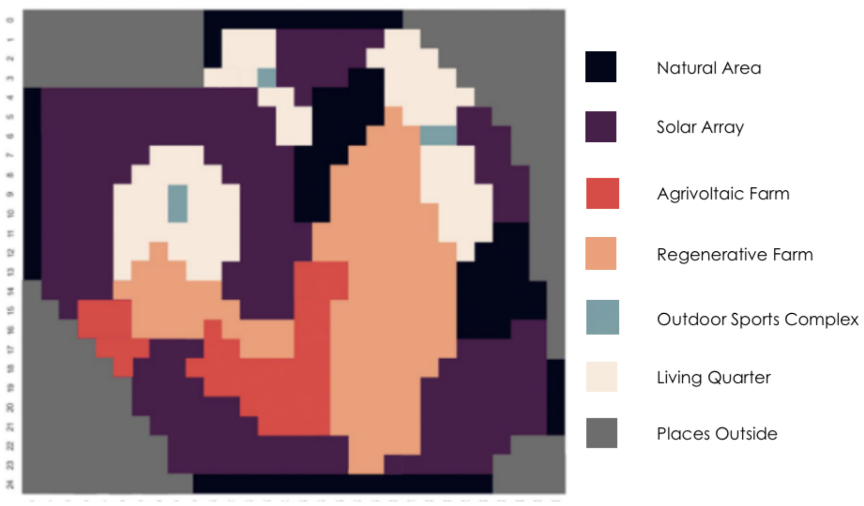


Figure 22 The Land Use Plan for Under-developed Region

In Figure 21 and 22, these are the land use plan for well-developed and under-developed regions.

7 Robustness Analysis

In order to test the stability of our model, we do a robustness analysis of our model. We will test whether our model will still provide a reasonable way of constructing facilities. We will provide an example for this robustness analysis:

We adjust the data of soil fertility into the region of $(0, 0.3)$(but the full mark is still 1), which means that this area is under severe drought, then we will test the solutions that our model give to us, the graphs are provided below:

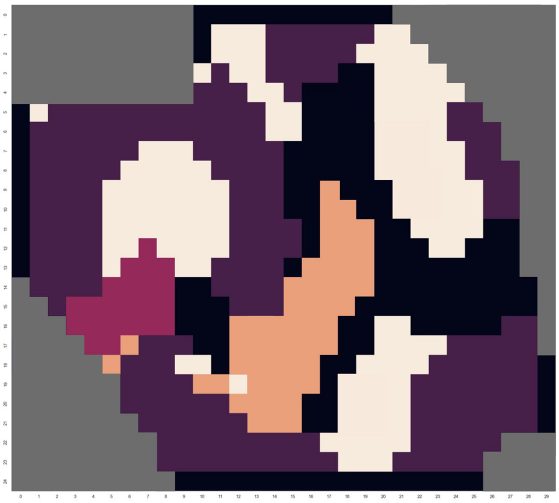


Figure 23 Original land use plan

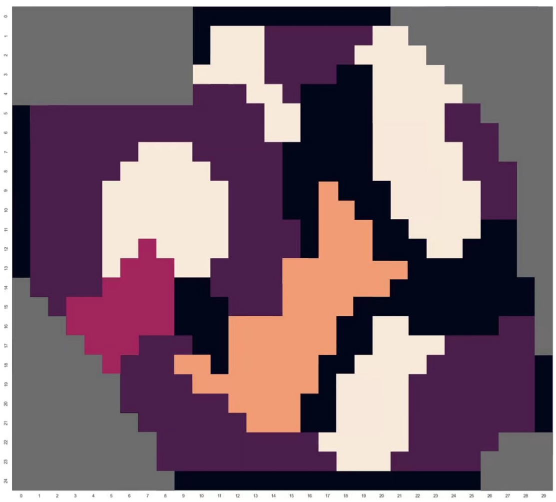


Figure 24 Actual land use plan

In Figure 23 and 24, these are the land use plan for low soil fertility before and after manual intervention.

Compared with the ones in former tasks, though there are some distinctions in facilities, but our land use planning model can still cover a high proportion of the land, with great favorabilities from residents and high expectation income for business investors. This means that our model is especially adaptive and stable even for extreme conditions.

8 Strengths and Weaknesses

8.1 Strengths

- The AHP methods we use provides a specific value for investors when considering different factors that are affecting the facilities. In addition, we have taken the most important factors such as money, environment, people's satisfaction into account, so our AHP results gives a highly reasonable measurement of the benefits of building these facilities.
- We have taken into account the measurement of geological factors as well as the different requirements for geological factors of different facilities. We demonstrated these requirements using different weights.
- We divided the maps into small sections, this makes our measurement of the facilities to build in this area highly accurate because every section will be considered in details about the facilities that should be built in these areas. And in the end, we have compiled all of these small areas into a big area to build a specific facility.
- We integrated both the results in the heat maps and the results in AHP. If a specific area is suitable for building two facilities, then we will choose the facility with the higher AHP value to ensure the benefits of investors. Therefore, we are confident that our model is both suitable for the geological conditions area and appealing to the investors.
- Our model is highly flexible. We can use our model for a variety of different lands by just making small changes in the geological data. This is because the value we calculated in AHP does not change with geographic attributes. Also, the requirements of different facilities over geological factors and the weights of them does not change either. So we only need to make some changes to the geological data and get a completely new result over what to build in a new land.

8.2 Weaknesses

Despite their superior performance, our model still has areas to improve:

- The geological data given by the map does not provide enough data for soil fertility and the humidity of the soil. So the data we collected may be inaccurate. To better enhance our model's performance and bolster the accuracy of our results, we should, in the future, use a wider range of data that offers more reliable and convincing conclusions.
- Due to the restriction of computer's calculating ability, we are not able to divide the whole areas into areas that are infinitively small. This means that the areas we divide may have variance in different data. In the future, we should divide the whole map into more areas to get more accurate conclusions.
- The factors we consider may not be enough for investors. Some investors may seek long-term profits, while others expect immediate income. In addition, some facilities are more durable than others, such as outdoor sports complex, making us unable to calculate their profit over time. In the future, we may take more factors into consideration and arrive at more accurate AHP results.

9 Conclusion

In our paper, we use AHP method to analyze the relative benefits of building different facilities, taking several important factors such as people's satisfaction and profits into consideration. We have also developed heat maps showing the relative appropriateness of the land in building a specific facility.

We use the integration of both of these methods to choose the facility to build in a specific land. Our model is able to determine the facility to build in a specific area according to both geological factors and the benefits of investors. Our model can also warn the investors about some areas that is not suitable for building any facility, so it can minimize the loss that will be brought if the facilities are built in that area. This model can be used to judge and optimize solutions for building facilities in the future.

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Model Designer

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Syracuse
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Syracuse, 13 March 2023

To community leaders and business planners

Letter of Syracuse Construction Plan

Through using the model, our team have found optimal land using plans both with the construction of the semiconductor facility and without it. In our plan, the land is divided into approximately ten parts, including living quarter, regenerative farm, solar array, and many other facilities. The exact plans can be seen in the figures in the essay.

Key Information of Our Decision Making

Specifically, the first step is to decide some facilities that is "good", which is a word with numerous definition. Either a facility that produce high income or a facility that is environmental friendly can be a good one.

Therefore, in order to compare all the facilities and choose some of the better ones, we need to use a model to develop a matric for decisions making. Instead of comparing a lot of optional facilities and their numerous pros and cons altogether, our model can divide this comparative analysis in multiple aspects into simple comparison, such as comparing the construction cost of regenerative farm and crop farm. Hence, we can consider every aspects objectively and comprehensively, without ignoring any parts of consideration. We thus developed the metric to evaluate different facility choices and find the relatively profitable, sustainable and conducive options according to our local conditions.

Then, after selecting some of the better options, we carefully analyze the geographical location, including altitude drop, electronic signals, plant cover and many other data and find out the specific areas in the land that are suitable for each facility. For the areas that are suitable for more than one kind of facility, we will choose the facility that has the highest metric value, meaning that this facility is better than others generally.

At last, note that there may be sporadical facility pieces, which will create difficulty for construction, we will then revise sporadical pieces into larger ones. Therefore, our plan can be both logical and clear.

Best Wishes!

IMMC Team #23021342

References

[1] Ross H. McKenzie PhD P. Ag., Understanding the effects of sunlight, temperature and precipitation, 22 November, 2017. https://www.topcropmanager.com/back-to-basics-20879/ Accessed 13 March 2023.

[2] Real Food, Real Farmers, Real Community℠. Altitude Matters to Crops. 09 Mar · Mon 2009. https://www.localharvest.org/blog/26102/entry/altitudematterstocrops Accessed 12 March 2023.

[3] Han Li, Mei Qiang, Ji min, et al. Analysis and Study on AHP-Fuzzy Comprehensive Evaluation. China Safety Science Journal, 2004, (7): 86 .

[4] Artem Milinchuk. Is Regenerative Agriculture Profitable? 30 Jan, 2020, 07:00am EST.https://www.forbes.com/sites/forbesfinancecouncil/2020/01/30/is-regenerative-agriculture-profitable/?sh=77882082cdf2 Accessed 9 March 2023.

[5] Georgette Kilgore. Solar Farm Income Per Acre Calculator: See Profit Margin, Costs, Money Made. 10 November, 2022. https://8billiontrees.com/solar-panels/solar-farm-income-per-acre/ Accessed 9 March 2023.

[6] Mbala, Mbuyamba, Clinton Aigbavboa, and John Aliu. ”Reviewing the negative impacts of building construction activities on the environment: The case of Congo.” Advances in Human Factors, Sustainable Urban Planning and Infrastructure: Proceedings of the AHFE 2018 International Conference on Human Factors, Sustainable Urban Planning and Infrastructure, July 21-25, 2018, Loews Sapphire Falls Resort at Universal Studios, Orlando, Florida, USA 9. Springer International Publishing, 2019.

[7] U.S. Department of Agriculture, Economic Research Service. Farm Sector Income Finances: Farm Business Income, 7 Feb. 2023. https://www.ers.usda.gov/topics/farm-economy/farm-sector-income-finances/farm-business-income/ Accessed 9 March 2023.

[8] JIANG Kun, DAI Wen-yuan, HU Qiu-feng, et al. Analysis on the Influence of Terrain Factors on Land Use Pattern in the Hilly Area of Zhejiang and Fujian Provinces: A Case Study of Yongtai County, Fujian Province. Journal of Ecology and Rural Environment, 2019, 35(6): 707-715.

[9] Alazzaz, Faisal, and Andrew Whyte. ”Uptake of off-site construction: benefit and future application.” International Journal of Civil and Environmental Engineering 8.12 (2014): 1219-1223.

Appendix A The matrices in AHP and their eigenvectors, CR value in task 1

Here are the six judging matrix of the eight facilities used in task 1, their CR value and their eigenvectors:

$$B_1 = \begin{bmatrix} 1 & 3/4 & 5 & 5 & 3 & 2 & 3/2 & 4 \\ 4/3 & 1 & 6 & 6 & 2 & 3/2 & 1 & 3 \\ 1/5 & 1/6 & 1 & 1 & 3/8 & 1/3 & 1/5 & 5/7 \\ 1/5 & 1/6 & 1 & 1 & 3/8 & 1/3 & 1/5 & 5/7 \\ 1/3 & 1/2 & 8/3 & 8/3 & 1 & 2/3 & 1/2 & 1 \\ 1/2 & 2/3 & 3 & 3 & 3/2 & 1 & 3/5 & 2 \\ 2/3 & 1 & 5 & 5 & 2 & 5/3 & 1 & 3 \\ 1/4 & 1/3 & 7/5 & 7/5 & 1 & 1/2 & 1/3 & 1 \end{bmatrix} \quad E_1 = \begin{bmatrix} 0.23 \\ 0.22 \\ 0.04 \\ 0.04 \\ 0.09 \\ 0.12 \\ 0.19 \\ 0.06 \end{bmatrix} \tag{12}$$

$$CR_{B_1} \approx 0.0076$$

$$B_2 = \begin{bmatrix} 1 & 1 & 1/3 & 1/3 & 1/5 & 1/5 & 1/4 & 1 \\ 1 & 1 & 1/3 & 1/3 & 1/5 & 1/5 & 1/4 & 1 \\ 3 & 3 & 1 & 1 & 1/3 & 1/3 & 1/2 & 3 \\ 3 & 3 & 1 & 1 & 1/3 & 1/3 & 1/2 & 3 \\ 5 & 5 & 3 & 3 & 1 & 1 & 2 & 5 \\ 5 & 5 & 3 & 3 & 1 & 1 & 2 & 5 \\ 4 & 4 & 2 & 2 & 1/2 & 1/2 & 1 & 4 \\ 1 & 1 & 1/3 & 1/3 & 1/5 & 1/5 & 1/4 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 0.04 \\ 0.04 \\ 0.10 \\ 0.10 \\ 0.25 \\ 0.25 \\ 0.16 \\ 0.04 \end{bmatrix} \tag{13}$$

$$CR_{B_2} = \frac{CI_{B_2}}{RI_{B_2}} \approx 0.0108$$

$$B_3 = \begin{bmatrix} 1 & 2/3 & 5 & 5 & 8/3 & 4/3 & 8/3 & 2/3 \\ 3/2 & 1 & 8 & 8 & 4 & 2 & 4 & 1 \\ 1/5 & 1/8 & 1 & 1 & 1/2 & 1/4 & 1/2 & 1/8 \\ 1/5 & 1/8 & 1 & 1 & 1/2 & 1/4 & 1/2 & 1/8 \\ 3/8 & 1/4 & 2 & 2 & 1 & 1/2 & 1 & 1/4 \\ 3/4 & 1/2 & 4 & 4 & 2 & 1 & 2 & 1/2 \\ 3/8 & 1/4 & 2 & 2 & 1 & 1/2 & 1 & 1/4 \\ 3/2 & 1 & 8 & 8 & 4 & 2 & 4 & 1 \end{bmatrix} \quad E_3 = \begin{bmatrix} 0.04 \\ 0.04 \\ 0.04 \\ 0.04 \\ 0.04 \\ 0.04 \\ 0.04 \\ 0.04 \end{bmatrix} \tag{14}$$

$$CR_{B_3} = \frac{CI_{B_3}}{RI_{B_3}} \approx 0.0000$$

$$B_4 = \begin{bmatrix} 1 & 2 & 1/3 & 1/3 & 1/3 & 4 & 2 & 1 \\ 1/2 & 1 & 1/4 & 1/4 & 1/4 & 3 & 1 & 1/2 \\ 3 & 4 & 1 & 1 & 1 & 6 & 4 & 3 \\ 3 & 4 & 1 & 1 & 1 & 6 & 4 & 3 \\ 3 & 4 & 1 & 1 & 1 & 6 & 4 & 3 \\ 1/4 & 1/3 & 1/6 & 1/6 & 1/6 & 1 & 1/3 & 1/4 \\ 1/2 & 1 & 1/4 & 1/4 & 1/4 & 3 & 1 & 1/2 \\ 1 & 2 & 1/3 & 1/3 & 1/3 & 4 & 2 & 1 \end{bmatrix} \quad E_4 = \begin{bmatrix} 1.34 \\ 2.04 \\ 0.26 \\ 0.26 \\ 0.51 \\ 1.02 \\ 0.51 \\ 2.04 \end{bmatrix} \tag{15}$$

$$CR_{B_4} = \frac{CI_{B_4}}{RI_{B_4}} \approx 0.0140$$

$$B_5 = \begin{bmatrix} 1 & 3 & 1/2 & 1/2 & 1 & 1/4 & 1/3 & 1 \\ 1/3 & 1 & 1/4 & 1/4 & 1/3 & 1/6 & 1/5 & 1/3 \\ 2 & 4 & 1 & 1 & 2 & 1/3 & 1/2 & 2 \\ 2 & 4 & 1 & 1 & 2 & 1/3 & 1/2 & 2 \\ 1 & 3 & 1/2 & 1/2 & 1 & 1/4 & 1/3 & 1 \\ 4 & 6 & 3 & 3 & 4 & 1 & 2 & 4 \\ 3 & 5 & 2 & 2 & 3 & 1/2 & 1 & 3 \\ 1 & 3 & 1/2 & 1/2 & 1 & 1/4 & 1/3 & 1 \end{bmatrix} \quad E_5 = \begin{bmatrix} 0.07 \\ 0.03 \\ 0.12 \\ 0.12 \\ 0.07 \\ 0.30 \\ 0.20 \\ 0.07 \end{bmatrix} \tag{16}$$

$$CR_{B_5} = \frac{CI_{B_5}}{RI_{B_5}} \approx 0.0117$$

$$B_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 \\ 2 & 2 & 2 & 2 & 2 & 1 & 1/2 & 2 \\ 4 & 4 & 4 & 4 & 4 & 2 & 1 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 \end{bmatrix} \quad E_6 = \begin{bmatrix} 0.67 \\ 0.67 \\ 0.67 \\ 0.67 \\ 0.67 \\ 1.13 \\ 2.67 \\ 0.67 \end{bmatrix} \tag{17}$$

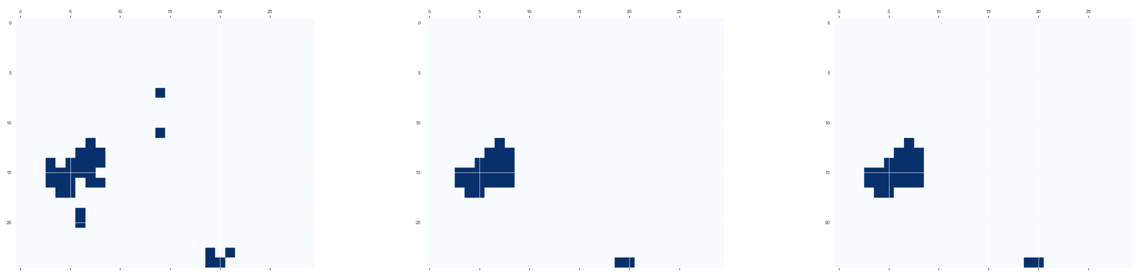
$$CR_{B_6} = \frac{CI_{B_6}}{RI_{B_6}} \approx 0.0000$$

The final calculations of the final vector:

$$\begin{bmatrix} 0.23 & 0.04 & 0.04 & 1.34 & 0.07 & 0.67 \\ 0.22 & 0.04 & 0.04 & 2.04 & 0.03 & 0.67 \\ 0.04 & 0.10 & 0.04 & 0.26 & 0.12 & 0.67 \\ 0.04 & 0.10 & 0.04 & 0.26 & 0.12 & 0.67 \\ 0.09 & 0.25 & 0.04 & 0.51 & 0.07 & 0.67 \\ 0.12 & 0.25 & 0.04 & 1.02 & 0.3 & 1.33 \\ 0.19 & 0.16 & 0.04 & 0.51 & 0.2 & 2.67 \\ 0.06 & 0.04 & 0.04 & 2.04 & 0.07 & 0.67 \end{bmatrix} \cdot \begin{bmatrix} 0.39 \\ 0.06 \\ 0.26 \\ 0.09 \\ 0.13 \\ 0.05 \end{bmatrix} = \begin{bmatrix} 0.2657 \\ 0.3196 \\ 0.1045 \\ 0.1045 \\ 0.149 \\ 0.2695 \\ 0.2995 \\ 0.2624 \end{bmatrix} \tag{18}$$

Appendix B Other block maps used in Task 2

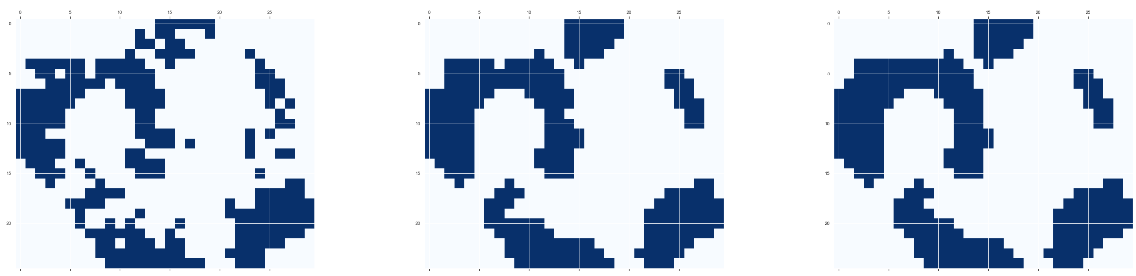
For cross-country skiing, we select the areas whose values are more than or equal to 0.6 and repeat the process for two times:



(a) Initial (b) First Operation (c) Second Operation

Figure 25 Selected areas of Cross-Country Skiing Facilities

For solar arrays, we will first select all the areas whose value is more or equal to 0.7.



(a) Initial (b) First Operation (c) Second Operation

Figure 26 Selected areas of Solar Arrays

Appendix C The new matrices used in Task 3

$$B_1' = \begin{bmatrix} 1 & 2 & 6 & 6 & 5 & 4 & 3 & 3 & 4/3 & 9/10 \\ 1/2 & 1 & 5 & 5 & 4 & 3 & 2 & 2 & 1 & 9/8 \\ 1/6 & 1/5 & 1 & 1 & 1/2 & 1/3 & 1/4 & 1/4 & 1/4 & 6/31 \\ 1/6 & 1/5 & 1 & 1 & 1/2 & 1/3 & 1/4 & 1/4 & 5/21 & 6/31 \\ 1/5 & 1/4 & 2 & 2 & 1 & 1/2 & 1/3 & 1/3 & 3/7 & 1/2 \\ 1/4 & 1/3 & 3 & 3 & 2 & 1 & 1/2 & 1/2 & 2/3 & 3/5 \\ 1/3 & 1/2 & 4 & 4 & 3 & 2 & 1 & 1 & 1 & 8/11 \\ 1/3 & 1/2 & 4 & 4 & 3 & 2 & 1 & 1 & 1 & 9/11 \\ 3/4 & 1 & 4 & 21/5 & 7/3 & 3/2 & 1 & 1 & 1 & 7/8 \\ 10/9 & 8/9 & 31/6 & 31/6 & 2 & 5/3 & 11/8 & 11/9 & 8/7 & 1 \end{bmatrix} \quad E_1' = \begin{bmatrix} 0.22 \\ 0.16 \\ 0.03 \\ 0.03 \\ 0.04 \\ 0.07 \\ 0.10 \\ 0.10 \\ 0.12 \\ 0.14 \end{bmatrix} \tag{19}$$

$$CR_{B_1'} \approx 0.0167$$

$$B_2' = \begin{bmatrix} 1 & 1 & 1/3 & 2/7 & 1/5 & 5/14 & 1/4 & 1 & 1/2 & 1/5 \\ 1 & 1 & 1/3 & 2/7 & 1/5 & 5/14 & 1/4 & 1 & 1/2 & 1/5 \\ 3 & 3 & 1 & 1 & 1/3 & 15/13 & 1/2 & 3 & 4/3 & 1/3 \\ 7/2 & 7/2 & 1 & 1 & 1/3 & 15/13 & 1/2 & 3 & 3/2 & 1/3 \\ 5 & 5 & 3 & 3 & 1 & 1 & 2 & 5 & 10/3 & 1 \\ 14/5 & 14/5 & 13/15 & 13/15 & 1/4 & 1 & 2 & 2 & 10/3 & 1/4 \\ 4 & 4 & 2 & 2 & 1/2 & 1/2 & 1 & 4 & 5/2 & 1/2 \\ 1 & 1 & 1/3 & 1/3 & 1/5 & 1/5 & 1/4 & 1 & 5/7 & 1/5 \\ 2 & 2 & 3/4 & 2/3 & 3/10 & 3/10 & 2/5 & 7/5 & 1 & 3/10 \\ 5 & 5 & 3 & 3 & 1 & 4 & 2 & 5 & 10/3 & 1 \end{bmatrix} \quad E_2' = \begin{bmatrix} 0.03 \\ 0.03 \\ 0.09 \\ 0.09 \\ 0.2 \\ 0.11 \\ 0.13 \\ 0.03 \\ 0.06 \\ 0.23 \end{bmatrix} \tag{20}$$

$$CR_{B_2'} \approx 0.0102$$

$$B_3' = \begin{bmatrix} 1 & 2/3 & 5 & 5 & 8/3 & 4/3 & 8/3 & 2/3 & 1/2 & 4/3 \\ 3/2 & 1 & 8 & 8 & 4 & 2 & 4 & 1 & 2/3 & 8/5 \\ 1/5 & 1/8 & 1 & 1 & 1/2 & 1/4 & 1/2 & 1/8 & 1/9 & 1/4 \\ 1/5 & 1/8 & 1 & 1 & 1/2 & 1/4 & 1/2 & 1/8 & 1/9 & 5/18 \\ 3/8 & 1/4 & 2 & 2 & 1 & 1/2 & 1 & 1/4 & 1/5 & 5/11 \\ 3/4 & 1/2 & 4 & 4 & 2 & 1 & 2 & 1/2 & 1/3 & 1 \\ 3/8 & 1/4 & 2 & 2 & 1 & 1/2 & 1 & 1/4 & 3/16 & 1/2 \\ 3/2 & 1 & 8 & 8 & 4 & 2 & 4 & 1 & 2/3 & 5/2 \\ 2 & 3/2 & 9 & 9 & 5 & 3 & 16/3 & 3/2 & 1 & 3 \\ 3/4 & 5/8 & 4 & 18/5 & 11/5 & 1 & 2 & 2/5 & 1/3 & 1 \end{bmatrix} \quad E_3' = \begin{bmatrix} 0.11 \\ 0.17 \\ 0.02 \\ 0.02 \\ 0.04 \\ 0.09 \\ 0.04 \\ 0.18 \\ 0.24 \\ 0.09 \end{bmatrix} \tag{21}$$

$$CR_{B_3'} \approx 0.0015$$

$$B_4' = \begin{bmatrix} 1 & 2 & 1/3 & 1/3 & 1/3 & 4 & 2 & 1 & 8/7 & 4/3 \\ 1/2 & 1 & 1/4 & 1/4 & 1/4 & 3 & 1 & 1/2 & 4/7 & 2/3 \\ 3 & 4 & 1 & 1 & 1 & 6 & 4 & 3 & 16/7 & 8/3 \\ 3 & 4 & 1 & 1 & 1 & 6 & 4 & 3 & 16/7 & 8/3 \\ 3 & 4 & 1 & 1 & 1 & 6 & 4 & 3 & 16/7 & 8/3 \\ 1/4 & 1/3 & 1/6 & 1/6 & 1/6 & 1 & 1/3 & 1/4 & 8/21 & 2/7 \\ 1/2 & 1 & 1/4 & 1/4 & 1/4 & 3 & 1 & 1/2 & 16/21 & 6/7 \\ 1 & 2 & 1/3 & 1/3 & 1/3 & 4 & 2 & 1 & 5/4 & 6/5 \\ 7/8 & 7/4 & 7/16 & 7/16 & 7/16 & 21/8 & 21/16 & 4/5 & 1 & 9/7 \\ 3/4 & 3/2 & 3/8 & 3/8 & 3/8 & 7/2 & 7/6 & 5/6 & 7/9 & 1 \end{bmatrix} \quad E_4' = \begin{bmatrix} 0.08 \\ 0.05 \\ 0.2 \\ 0.2 \\ 0.2 \\ 0.02 \\ 0.05 \\ 0.08 \\ 0.07 \\ 0.07 \end{bmatrix} \tag{22}$$

$$CR_{B_4'} \approx 0.0099$$

$$B_5' = \begin{bmatrix} 1 & 2 & 1/3 & 1/3 & 1/2 & 1/4 & 1/2 & 1 & 1 & 2 \\ 1/2 & 1 & 1/4 & 1/4 & 1/3 & 1/5 & 1/3 & 1/2 & 1/2 & 1 \\ 3 & 4 & 1 & 1 & 2 & 1/2 & 2 & 3 & 3 & 4 \\ 3 & 4 & 1 & 1 & 2 & 1/2 & 2 & 3 & 3 & 4 \\ 2 & 3 & 1/2 & 1/2 & 1 & 1/3 & 1 & 2 & 2 & 3 \\ 4 & 5 & 2 & 2 & 3 & 1 & 3 & 4 & 4 & 5 \\ 2 & 3 & 1/2 & 1/2 & 1 & 1/3 & 1 & 2 & 2 & 3 \\ 1 & 2 & 1/3 & 1/3 & 1/2 & 1/4 & 1/2 & 1 & 1 & 2 \\ 1 & 2 & 1/3 & 1/3 & 1/2 & 1/4 & 1/2 & 1 & 1 & 2 \\ 1/2 & 1 & 1/4 & 1/4 & 1/3 & 1/5 & 1/3 & 1/2 & 1/2 & 1 \end{bmatrix} \quad E_5' = \begin{bmatrix} 0.06 \\ 0.03 \\ 0.16 \\ 0.16 \\ 0.10 \\ 0.25 \\ 0.10 \\ 0.06 \\ 0.06 \\ 0.03 \end{bmatrix} \tag{23}$$

$$CR_{B_5'} \approx 0.0083$$

$$B_6' = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 1 & 1/2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 & 4 & 2 & 1 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 & 1/4 & 1 & 1 & 1 \end{bmatrix} \quad E_6' = \begin{bmatrix} 0.07 \\ 0.07 \\ 0.07 \\ 0.07 \\ 0.07 \\ 0.14 \\ 0.29 \\ 0.07 \\ 0.07 \\ 0.07 \end{bmatrix} \tag{24}$$

$$CR_{B_6'} \approx 0.0000$$

In the matrix above, B_i' means the judging matrix of factor C_i after the factory is built and E_i' means the eigenvector of matrix B_i'. In addition, all the CR values are below 0.1, meaning that these matrix are logical and the eigenvectors are effective.

For the judging matrix B_1', the relative income for $P_1 P_2$, and P_8 will increase. Since more people will be around, a larger amount of them will visit those entertainment facilities. This will thus create more income for outdoor sports complex, cross-country skiing facility and agritourist center.

As for matrix B_5', the maintenance cost will be higher for modern artificial facilities(P_1, P_2), since more people will be here and the facilities will burn-in more quickly. Besides, the maintenance cost of other kinds of facilities, such as farms and solar arrays, will just increase a little because of higher employers' salaries result from the salary(for about 100,000 dollars) of the newly-constructed fab.

Again using AHP, we calculate the eigenvectors of these matrix using Python, which are also shown above.

The calculations are similar to the one shown in appendex 1.

Appendix D Proof of point F in the convex polygon

We consider the whole living area in a complex plane, and the complex number representing A_i is a_i. We only need to prove that for a point F' that satisfy $\angle A_i F A_{i+1} = \frac{2\pi}{n}$ and another point P inside the convex polygon satisfy(P's complex number is z):

$$\sum_{i=1}^{n} PA_i \geq \sum_{i=1}^{n} F'A_i \tag{25}$$

Note that because for any i, $\angle A_i F' A_{i+1} = \frac{2\pi}{n}$, so if take the n-th unit root ϵ, then there must be a t that satisfy $a_k \epsilon^t \in R$. Note that there is no particular order of t, so we can assume $a_k \epsilon^k \in R$.

Therefore, since $|\epsilon^k| = 1$:

$$RHS = \sum_{i=1}^{n} |F'A_i| \tag{26}$$

$$= \sum_{i=1}^{n} |k| \cdot |\epsilon^k| \tag{27}$$

$$= |\sum_{i=1}^{n} a_k \cdot \epsilon^k| \tag{28}$$

$$\begin{aligned}
LHS &= \sum_{i=1}^{n} |PA_i| && (29)\\
&= \sum_{i=1}^{n} |a_k - z| && (30)\\
&= \sum_{i=1}^{n} |(a_k - z) \cdot \epsilon^k| && (31)\\
&\geq |\sum_{i=1}^{n} (a_k - z) \cdot \epsilon^k| && (32)\\
&= |\sum_{i=1}^{n} a_k \cdot \epsilon^k - z \cdot \sum_{i=1}^{n} \epsilon^k| && (33)\\
&= |\sum_{i=1}^{n} a_k \cdot \epsilon^k| = RHS && (34)\\
& && (35)
\end{aligned}$$

Therefore, we have proven that point F satisfy this inequality.This means that F is the same point with F and we can find point F using $\angle A_i F A_{i+1} = \frac{2\pi}{n}$. Note that this point F is also called as Fermat's point in mathematics.