Team Control Number

12733

Problem Chosen

A

2022
HiMCM
Summary Sheet

Nowadays, beekeepers are deeply concerned with the possibility of the collapse of bee colonies due to issues with viruses and pesticides. Beekeepers also try to increase the productive output of flowers by placing beehives in optimal locations.

In response to the first question, we decided to build a series of differential equation depicting the number of bees over time. In order to solve the second challenge,we created an algorithm that depicts the number of bees and we design a programming model that can indicate positions of the bee hives to maximize the profit that bee keepers can get.

The first part of our model establishes a differential equation of the number of bees in the colony .We divide the bee's life into different stages and carefully build equation of the number of each stage. The stages include unfertilized eggs, fertilized eggs, hive bees and foragers.We take many factors such as the rate of fertilization of eggs, the eclosion rate, the virus that threaten the bees, the pesticides,etc into consideration and develop a comprehensive model of the total number of the bees. We also successfully calculated the static numeric value of the bee colony as well as the maximum mortality rate to warn the farmers of the possibility when the bee colony will collapse.

The second part of our model, we analyze the sensitivity of 5 variables that we calculated in our model in the first part using some common data from bee farms. We demonstrated how these variables affect the colony size using detailed graphs and scientific analysis.We concluded that the capacity of the bee hive, the production rate of the eggs and the fertilized rate have positive effect on the bee colony, while the mortality rate has negative effect.

The third part of our model we used mathematical expectation algorithm to concisely depict the complicated tendency of bees visiting flowers. We use programming model to maximize the profit (the money bee keepers get from flowers that successfully grow food-the cost of managing the bee hives) bee keepers will receive by operating on the coordinate of the bee hives as well as the number of the bees in each of the bee hives according to the number of flowers and environmental conditions in the farm. Using this algorithm, farmers can get an idea of where to put their bee hives and the number of the bees for each hive.

We also designed a information graphic page to demonstrate the data that we get from our model vividly to others.

Keywords:

differential equation; critical point of mortality; mathematical expectation; optimal profit

Hive bees

Team # 12733

Introduction

Honeybees play a paramount role to human existence on our planet. In 2007, the term Colony Collapse Disorder (CCD) was created to describe the decline of honeybee populations around the world.

Our vision

In the paper, we'll analyze what affects its population and develop models, using differential equations, to determine the population of a honeybee colony over time. We make models and predict how many honeybee hives we will need to support pollination of a 20-acre (81,000 square meters) parcel of land containing crops that benefit from pollination by using the programming model.

Processes

The first model:

We divide the change rate into the natality and the mortality, as the former is made up of fertilization rate and hatching rate and the latter is result from four elements: anthropic factors, predators, viruses, and natural ages.

The second model:

We use the progrmming model to find out how many beehives and where beehives locate maximize the profit.

Contents

1 Introduction **1**

1.1 Background . . . 1

1.2 Problem Restatement . . . 1

2 Assumptions and Justification **1**

3 Variables **2**

4 Colony Demographic Model **4**

4.1 Differential Equation Analysis . . . 4

4.1.1 Model framework . . . 4

4.1.2 The Unfertilized Eggs . . . 5

4.1.3 The Fertilized Eggs . . . 5

4.1.4 Foragers and Hive Bees . . . 6

4.1.5 Mortality Rate . . . 7

4.2 Stable Analysis . . . 8

4.3 Numerical Experiment . . . 9

4.4 Sensitivity Analysis . . . 10

5 Pollination Programming **12**

5.1 Mathematical Model of Optimal Profit . . . 12

5.2 Algorithm Design . . . 15

5.3 Numerical Experiment . . . 15

5.4 Sensitivity Analysis . . . 17

6 Evaluation **19**

Appendices **20**

Appendix A The Globally Stable Steady State **20**

1 Introduction

1.1 Background

Bees are vital to the environment [1]. The most important function they serve is the pollinate our plants, meaning they carry pollen from plants of different sex, helping them reproduce. Bee colonies are often used by farmers to produce honey, producing more than 1.6 million tons of honey annually with 80 million hives. They are also essential to maintaining biodiversity in an ecosystem.

However, the population of bee colonies is declining globally. For over a decade, habitat loss, extensive farming practices, and climate change have all threatened the bee population. Furthermore, wide-scale transport of bees may result in the transmission of pathogens, causing more harm to the bees. Reducing the rate at which these factors occur would stop the decline in bee population [2].

1.2 Problem Restatement

The population of a honeybee colony is decided by many complicated factors. The first question asks us to predict the population of a honeybee colony. Here, we create a model of the honeybee colony in order to measure the growth of its population over time. It considers only the most important factors, such as season, making it a simplified version of its natural counterpart.

Apart from the population, profit is another key factor to consider for farmers. Therefore, the second problem asks how many beehives are required to support a plot of land with an area of 20 acres. To solve this problem, we need to consider the amount as well as the placement of the beehives, since different placements can result in vastly different results. The model uses iteration to calculate both variables. The profit is then calculated to rank the results.

2 Assumptions and Justification

1. Assumption: The brood reared by the queen bee is only affected by the season and the remaining capacity of the hive [3].

   Justification: In nature, the queen bee usually lay eggs according to the remaining capacity of the hive, when there are few spaces left for new eggs, the queen bee will reduce her rate of laying eggs. Seasons have to be taken into consideration too, because the queen bee tend to lay more eggs in the summer but fewer in the winter.

2. Assumption: We ignore the effect of drone fertilization, we can ignore the population of drones.

   Justification: In real life, unfertilized eggs will eventually become drones. However, the fertilized rate of the eggs actually depend on the number of the drones which makes the process really complicated. To simplify this process we define the fertilized rate as a variable that only depends on the seasons, and thus ignore the effect of the drones.

3. Assumption: Hive capacity does not include foragers.

   Justification: Foragers mainly collect honey outside, so we can assume that the hive bee capacity do not include foragers.

4. Assumption: Ignore the recovery of sick bees.

Justification:Since the death rate is mainly determined by the bees' living conditions, we will set it as a constant.

5. Assumption: Mimicking how organisms interact in nature, we will set the predatory ability of predators α on bees as constant C.

6. Assumption: The natural life span of a bee is determined only by the pollen density and its working conditions.In addition,the amount of work a bee does is only determined by the season.

   Justification: The pollen density can be seen as the food source that sustain the bee's growth. Bees tend to work harder during the summer, resulting in a sharper decline in their population. We therefore have to take the season and the working amount of bees into consideration.

7. Assumption: The destruction done by humans to bees is always faster than the regeneration speed of the bees' environment.

   Justification: This is due to the common fact that the environmental damage human caused is really severe and this will often cause damage that the natural environment is not able to recover.

8. Assumption: Farmers only plant 1 type of flower.

   Justification: Farmers tend to plant only 1 type of plant in a single area to avoid unnecessary competition among the flower species.In addition, this assumption can simplify our estimation.

9. Assumption: Ignore the time it takes for bees to go from a fertilized egg to an adult.

   Justification: We only want to calculate the increasing rate of the hive bees and foragers. Therefore, we will ignore this process and try to calculate this process by assessing the bee colony as a whole without considering a single bee's growth.

10. Assumption: Bees will not take into account the decision of other bees when selecting their flower, which means that the flower selection of bees is only determined by distance and the number of bees will not change with time.

    Justification: In real life situation, the bees' main goals are getting the pollen from the flowers, so we can ignore the competition among bees to simplify our results. Also, farmers tend to take good care of their bees, so we suppose that the number of bees remain unchanged in this period.

11. Assumption: The total distance traveled by the bee will never surpass its maximum flying distance.

    Justification: Bees are able to fly and work in a relatively large distance. So to simplify the results, we will assume that the total distance will not succeed the bee's limit.

3 Variables

Variables	Description
B	the number of unfertilized eggs
A	the number of fertilized eggs
H	the number of hive bees
F	the number of foragers
β	the rate of fertilizing
R	the capacity of the bee hives
pn	the number of predators
a	the ability that predators catch the bees
S_1, S_2, S_3, S_4	the spring ,summer ,autumn, winter egg production rate of the queen bee
S	the brooding speed
t	the mortality rate of the virus
r	the natural death rate of the bees
φ	the loss ratio of the unfertilized eggs
E	the workload of the bee
E_0	the lowest workload of the bee
E_1, E_2, E_3, E_4	the spring, summer, autumn, winter workload
ρ	the maximum density of the pollen
v	the overall decreasing rate of the pollen density
ε	the number of times that the flowers need to be visited by the bees in order to get fruit
N	the number of flowers
$T_{i,j}$	the distance of the ith bee hive and the jth flower
A	the total number of the fruited flowers
(x_i, y_i)	the coordinate of the ith bee hive
N_i	the number of bees in the ith bee hive
(n_j, m_j)	the coordinate of the jth flower
Δ_S	the expected number of the times that the Sth flower that will be visited
q	the cost of installing one bee hive
r	the cost of caring for one bee in this period
Z	the profit that the bee keepers will get when a single flower grow fruit
P	the total profit that the bee keeper will get
l	the number of bee hives
m	the death rate of bees
d	the natural contribute factor of the bee's death rate
p	the pesticides factor
$p_{1,2,3,4}$	the spring, summer, autumn, winter pesticides killing rate
k	the passing time
t	the factor of viruses

Table 3.1: Model variables and description

4 Colony Demographic Model

4.1 Differential Equation Analysis

4.1.1 Model framework

Firstly, we will briefly summarize the framework of whole bee colony (shown in Figure 4.1). There is one queen bee that produces eggs. Unfertilized eggs are produced and the eggs which are fertilized will turn into hive bees. The ones who are not fertilized will turn into drones or get lost.

Then, the fertilized eggs will receive food given by the hive bees and there is a eclosion rate of fertilizing eggs turning into hive bees.

Foragers is a kind of bee that finds food outside the bee hive and usually do not stay in the bee hive for a long time. Therefore, foragers will be more likely to died because of outside dangers such as predators, viruses, old age.

The colony will consistently monitors the balance between the number of hive bees and the foragers.Once foragers are in a relatively low number, some of the hive bees will transform into foragers. The process of the opposite direction can also occur, with the number of hive bees decreasing and foragers turning into hive bees.The number of bees in the colony that we want to calculate is the sum of the number of hive bees and foragers.

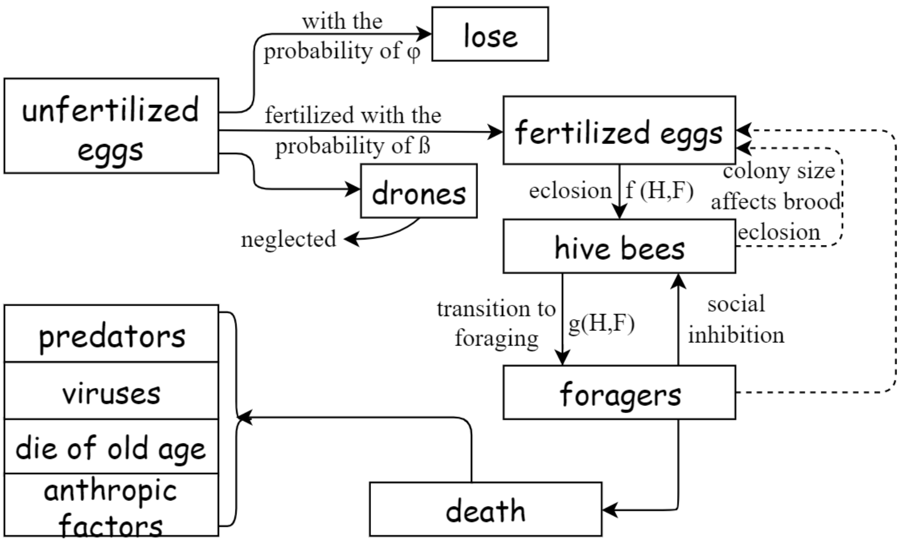

Figure 4.1: process

Since we hypothesize that the hive only has one queen bee, we only need to consider the unfertilized eggs that it lays. Unfertilized eggs may have the probability of β turning into fertilized eggs, while others will grow into drones, which we will ignore. Moreover, the unfertilized eggs will lose with the probability of φ.

We hypothesise the amount of brood reared is influenced by the size of the colony (number of hive and forager bees),the relation of which can be represented as $f(H, F)$. Additionally, that the rate at which bees transition from hive bees to foragers is influenced by the number of foragers to represent the effect of social inhibition, the connection of which can be illustrated as $g(H, F)$.

Our model considers the death rate of adult bees within the hive to be negligible, but foragers' death rate is a parameter varied in our simulations. In the model, we hypothesise that the death of foragers only result from four elements: predators, viruses, natural death and human influence factors.

4.1.2 The Unfertilized Eggs

We first consider the change rate of the number of unfertilized eggs using differential equations. It can be perceived that the speed that the queen bee produces eggs, which can be called brooding speed as well, may affect the change rate of unfertilized eggs.

The brooding speed S satisfies this equation:

$$S = \lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3 + \lambda_4 S_4, \tag{4.1}$$

where $\lambda_i \in 0, 1$ for $i \in [1, 2, 3, 4]$. In this equation, S_i represents the brooding speed of different seasons. For example, $\lambda_3 = 1$ while $\lambda_1, \lambda_2, \lambda_4 = 0$ when it's autumn. In addition, S normally is 2000.

Similarly, the fertilization rate β is $\lambda_1\beta_1+\lambda_2\beta_2+\lambda_3\beta_3+\lambda_4\beta_4$,where β_i represents the seasonal fertilizing rate. Since fertilizing means that the unfertilized eggs will turn into fertilized eggs, we need to minus this factor.

Moreover, it should be found that some of the unfertilized eggs may be lost or elosed into drones, we define φ as the loss rate.

We also have to consider the effect of the remaining bee hive capacity $R - A - B - H$. The brooding speed will be affected by the ratio of the remaining space and the whole space of the bee hive.

Therefore, we obtain this equation:

$$\frac{dB}{dt} = S\frac{R - A - B - H}{R} - \beta B - \varphi B \tag{4.2}$$

where R is the capacity of the bee hive, normally R's value will range from 20000 to 80000.

4.1.3 The Fertilized Eggs

The next step we will find the differential equation about the number of fertilized eggs.

We will first try to analyze the successful rate in which the eggs turn into hive bees.Nutrient quality w may affect the success rate of fertilized eggs hatching into hive bees, so we use $f(H, F)$,where H is the number of hive bees and F is the number of foragers, to indicate the success rate.

We need to note that both the foragers and the hive bees are really important in the feeding of the fertilized eggs since in nature the foragers find food and the hive bees turn the food into materials that the eggs can absorb, so the number of both of them are positive factors to f(H,F).

Therefore, we define:

$$f(H, F) = \frac{H + F}{\omega + H + F} \tag{4.3}$$

From Figure 4.2, we can see the relationship between of H and F in different ω. ω is lower when the nutrient level of a certain place is higher,we can see that when ω is lower, the rate which $f(H, F)$ change with the value of $H + F$ decreases.

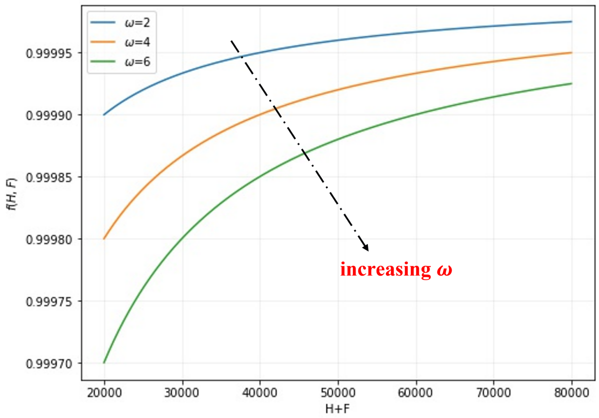

Figure 4.2: The graph between $H + F$ and $f(H, F)$ with different values of ω

We recall that the fertilized eggs will increase with a speed of βB,which we have already calculated in the previous section

Now we are able to calculate the number of fertilized eggs by integrating both the increase and the decrease in the number of fertilized eggs.The equation can be indicated as:

$$\frac{dA}{dt} = \beta B - f(H, F)A \tag{4.4}$$

4.1.4 Foragers and Hive Bees

Then, we are supposed to find the differential equation about the number of hive bees $H$and foragers F.

We assume that the death rate of hive bees is negligible because death rates of adult hive bees in a healthy colony are extremely low as the environment is protected and stable.

Workers are recruited to the forager class from the hive bee class and we define the recruitment rate as:

$$g(H, F) = (\alpha - \sigma\frac{F}{H + F}) \tag{4.5}$$

note that the recruitment that we assume is from H to F, therefore, if the value we have calculated is positive, it means that hive bees are converting to foragers, if the value we have calculated is negative, it means that foragers are converting into hive bees

As for $g(H, F)$, we use α to denote the maximum rate that hive bees will become foragers when there are no foragers present in the colony.

The second factor we need to concern can be illustrated as

$$-\sigma\frac{F}{H + F} \tag{4.6}$$

which represents social inhibition and how the presence of foragers reduces the rate of recruitment of hive bees to foragers. This equation depicts the common fact that when the foragers are a large amount of the bee colony population, the eclosion rate tends to be low, σ represents local issues

We learn from reference [2] that in the absence of any foragers new workers will become foragers at a minimum of four days after eclosing, so an appropriate choice for the rate of uninhibited transition to foraging is α=0.25. We chose σ=0.75 since this factor implies that a reversion of foragers to hive bees would only occur if more than one third of the hive are foragers.

We have assumed that social inhibition is directly proportional to the fraction of the total number of adult bees that are foragers, such that a high fraction of foragers in the colony results in low recruitment.

According to the formal computes, we can find the differential equation about the number of hive bees and foragers:

$$\frac{dH}{dt} = f(H,F)A - g(H,F)H \tag{4.7}$$

$$\frac{dF}{dt} = g(H,F)H - mF \tag{4.8}$$

where m is the mortality rate of the foragers, since we ignore the death of other bees.

4.1.5 Mortality Rate

After that, we ought to calculate the death rate of foragers m. We have assumed that only four factors—-predators, viruses, pesticides and dying of natural age, and the calculation process of which will be indicated as follows.

Firstly, We assume that the predation quantity of predators only depends on their hunting ability a and their population pn, so its effect can be represented as

$$\varphi(pn) = ap \tag{4.9}$$

Secondly, as regards the impact of viruses, infected foragers can't recover in our model, therefore we use t to show the impact.

Thirdly, as for the pesticides, since the amount of pesticides using depends on the season, it can be seen as

$$p = \lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3 + \lambda_4 p_4 \tag{4.10}$$

where p_i represents the seasonal effect of pesticides, we defined similar variables before.

Finally, bees' natural length of life may be affected by their workloads E and the environment. The effect of the environment on bee's natural death is really complex. Therefore, we only consider the density of flowers ρ as a direct environmental factor.

The caseloads E can be indicated as

$$E = \lambda_1 E_1 + \lambda_2 E_2 + \lambda_3 E_3 + \lambda_4 E_4 \tag{4.11}$$

and the density of flowers ρ can be demonstrated as

$$\rho = \rho_0 - vk \tag{4.12}$$

where ρ_0 means the highest density of flowers,and v means the speed of human destroying flowers minus the speed of flowers growing, k is the pasting time.

We also realize that the work load that exceeds the lowest work load E_0 has negative effect on the bee's natural death rate. So the factor is defined as:

$$d = \frac{T}{\rho}\left(1 - \frac{\theta}{E - E_0}\right). \tag{4.13}$$

where T and θ contributes to local and environmental conditions.

Therefore, m can be represented as:

$$m = \varphi(pn) + t + p + d \tag{4.14}$$

detailing the equation we get :

$$m = ap + \lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3 + \lambda_4 p_4 + t + \frac{T}{\rho_0 - vk}\left(1 - \frac{\theta}{\lambda_1 E_1 + \lambda_2 E_2 + \lambda_3 E_3 + \lambda_4 E_4 - E_0}\right) \tag{4.15}$$

4.2 Stable Analysis

Finally, in order to warn the bee keepers about the stability of the bee colony. We will try to calculate the numerical value when the hive bees and the foragers are stable. We calculate this value in order to calculate the maximum mortality rate m. We calculate this by letting all of the differential equation's value=0 and trying to solve this equation:

$$\frac{dH}{dt} = 0; \frac{dF}{dt} = 0; \frac{dA}{dt} = 0; \frac{dB}{dt} = 0 \tag{4.16}$$

The model has a globally stable steady state $(\hat{H}, \hat{F})$ where

$$\hat{F} = \frac{MR - \frac{Mm\omega}{1+\frac{1}{\lambda}}}{Rm + \frac{R\varphi m}{\beta} + \frac{M}{\lambda} + \frac{Mm}{\beta} + Mm} \tag{4.17}$$

$$\hat{H} = \frac{MR - \frac{Mm\omega}{1+\frac{1}{\lambda}}}{\lambda(Rm + \frac{R\varphi m}{\beta} + \frac{M}{\lambda} + Mm)} \tag{4.18}$$

where we regard $\lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3 + \lambda_4 S_4$ as M and $\frac{F}{H}$ as λ and

$$\lambda = \frac{-(m + \sigma - \alpha) + \sqrt{(m + \sigma - \alpha)^2 + 4m\alpha}}{2m} \tag{4.19}$$

We need to mention that if m makes the stable number of both the hive bees and the foragers under zero, then obviously the bee colony will die out. The conditions for $\hat{F}$ and $\hat{H}$ to be larger than 0 is:

$$MR > \frac{Mm\omega}{1 + \frac{1}{\lambda}} \tag{4.20}$$

We therefore get the critical point of mortality:

$$m^2(\alpha\omega - R) - m(2m\alpha + R\sigma - R\alpha) + \frac{\sigma R^2}{\omega} < 0 \tag{4.21}$$

The whole process can be found in Appendix.

We can see from this equation. The stable number of the hive bees as well as the foragers rely on the bee hive capacity R, the seasonal production of eggs S, the lose rate of the unfertilized eggsφ, the mortality rate m, the relative stable ratio of λ the fertilizing rate β,local issues, such as σ and the local nutrient rateω as well as the maximum rate of turning into foragers α.

We can also conclude that only if m satisfy the inequality we get that the bees are not going to die. This equation consists of variables that are determined by local issues, such as σ and the local nutrient rate ω,but it is also determined by the capacity of the hive R, the maximum rate of turning into foragers α.

4.3 Numerical Experiment

We run numerical experiment for the designed demographic model, the detailed parameters and corresponding value is given in Table 4.1.

Parameters	R	B initial	A initial	H initial	F initial	β	α
Values	0.6	0.1	2000	0.8	0.85	0.8	0.7
Parameters	φ	m	S_1, S_2, S_3, S_4	L_1	L_2	L_3	L_4
Values	80000	1000	1000	15000	5000	0.5	0.25

Table 4.1: Model Parameters

Figure 4.3 shows the relationship between the number of fertilized and unfertilized eggs with time. Both values show little relation to the time of year. Rather, the values remain mostly constant, with the total population stabilizing at around 2000 [5]. However, one general trend that remains constant is that the amount of unfertilized eggs is always larger than the amount of fertilized eggs.

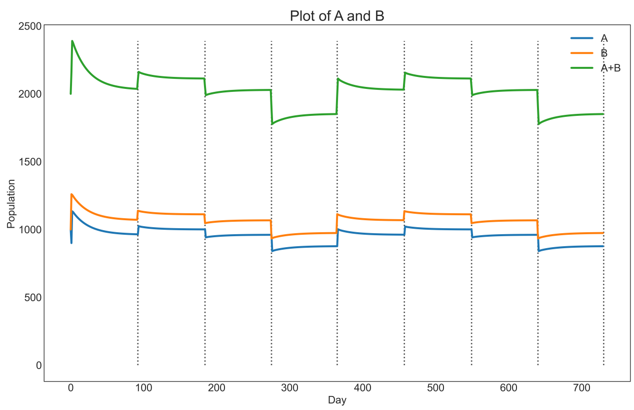

Figure 4.3: Fertilized eggs (A) and unfertilized eggs (B) against time

Figure 4.4 shows the relationship between the number of hive bees and foragers with time. Both values have increased from the initial value. All values rise sharply at the start, with the total going from around 20000 to 35000. After that, the population remains mostly constant, save some small fluctuations and one significant drop at around day 300. The total population at day 300 400 dropped from around 33000 to 31000. Since day 300 is around the time which Winter starts, the weather may explain this sudden decline. However, the population quickly recovers.

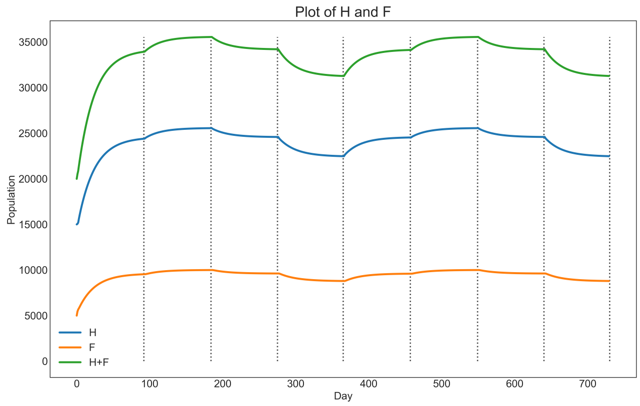

Figure 4.4: Hive bees (H) and foragers (F) against time

4.4 Sensitivity Analysis

The following Table 5.1 displays the range for which each variable was tested. Variables not included in the table remain the same as in the last table. Figure 4.5 and Figure 4.6 show

Parameters	Range	Amount generated
S	$1000 \leq S \leq 3000$	100
m	$0 \leq m \leq 1$	100
β	$0 \leq \beta \leq 1$	100
R	$20000 \leq R \leq 80000$	100
φ	$0 \leq \varphi \leq 1$	100

Table 4.2: Parameter Range

a sensitivity analysis on m. For Figure 4.5, it can be seen that an increase in m causes the total population to decrease. The effect is most noticeable at smaller values. Likewise, for Figure 4.6, an increase in m is also reflected in a decrease in the overall population. However, fluctuations in the population from changes of season can also be noticed, as shown through a decrease in population every time winter comes (3 times, since we ran the simulation for 1095 days = 3 years).

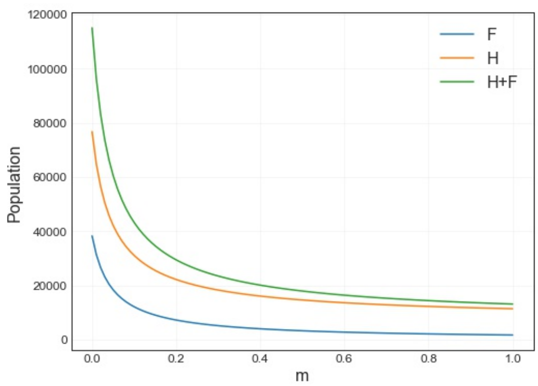

Figure 4.5: Relationship between the death rate (m) to population

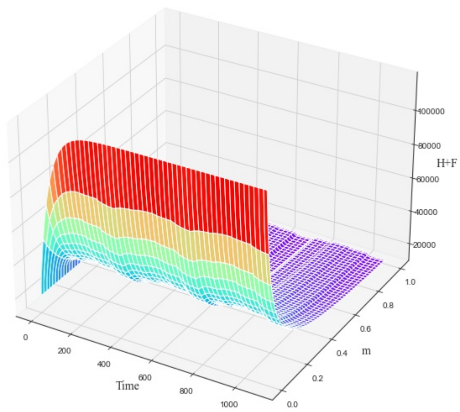

Figure 4.6: Relationship between the death rate (m) to population and time

Figure 4.7 and Figure 4.8 shows a sensitivity analysis on β. For Figure 4.7, it can be seen that an increase in β causes an increase in the total population. Likewise, for Figure 4.8, an increase in β is also reflected in an increase in the overall population. Very tiny fluctuations in the population from winter coming are also shown (3 times, since we ran the simulation for 1095 days = 3 years).

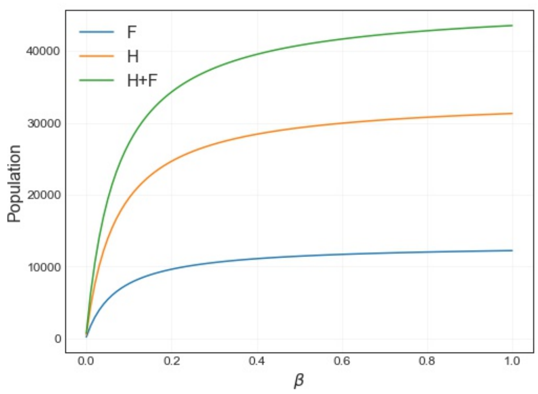

Figure 4.7: Relationship between the rate of fertilization (β) to population

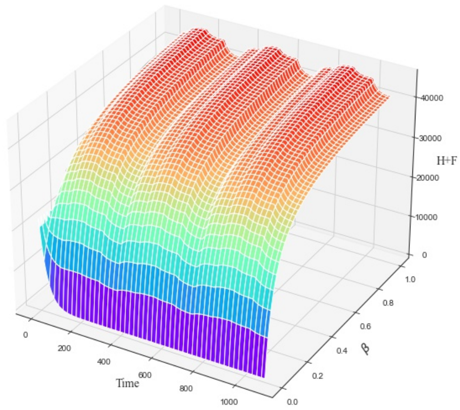

Figure 4.8: Relationship between the rate of fertilization (β) to population and time

Figure 4.9 and Figure 4.10 show a sensitivity analysis on S. For Figure 4.9, it can be seen that an increase in S causes an increase in the total population. Likewise, for Figure 4.10, an increase in S is also reflected in an increase in the overall population. However, large fluctuations in the graph can be seen from the season becoming winter.

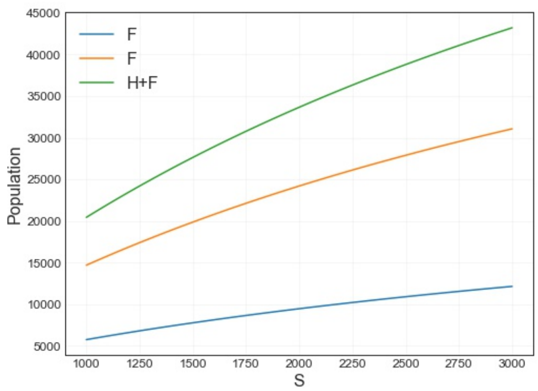

Figure 4.9: Relationship between the brooding speed (S) to population

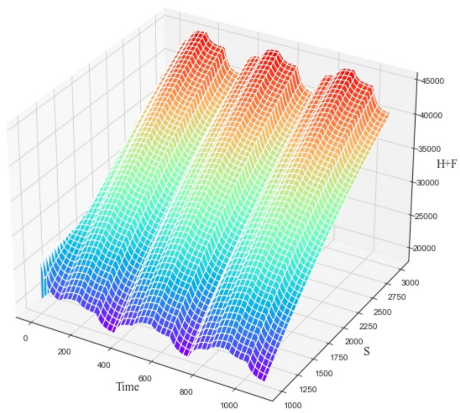

Figure 4.10: Relationship between the brooding speed (S) to population and time

The trend of R, as shown through Figure 4.11 and Figure 4.12, is the same as variable R. An increase in R causes an increase in the total population.

Figure 4.13 and Figure 4.14 show a sensitivity analysis on φ. For Figure 4.13, it can be seen that an increase in φ causes a decrease in the total population. Likewise, for Figure 4.14,

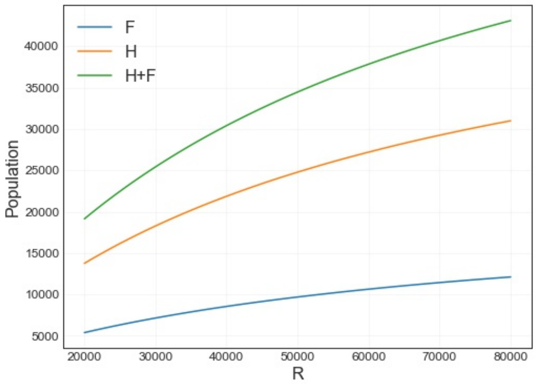

Figure 4.11: Relationship between the hive capacity (R) to population

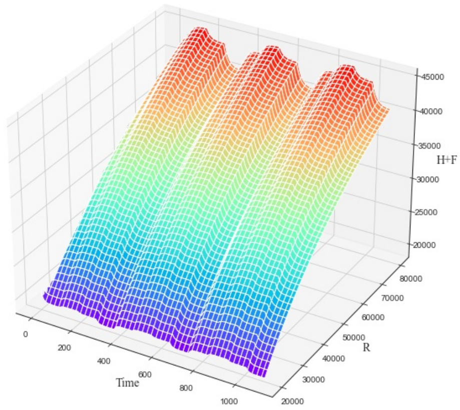

Figure 4.12: Relationship between the hive capacity (R) to population and time

an increase in φ is also reflected in a decrease in the overall population. Large fluctuations are also visible.

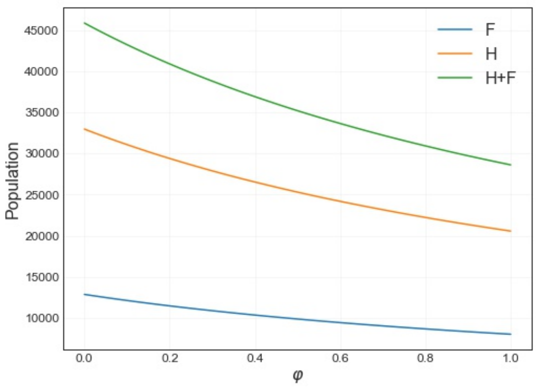

Figure 4.13: Relationship between the loss ratio of the unfertilized eggs (φ) to population

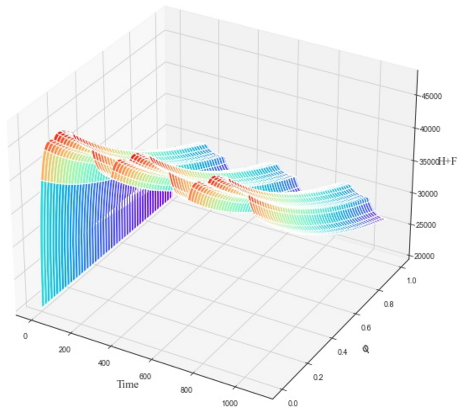

Figure 4.14: Relationship between the loss ratio of the unfertilized eggs (φ) to population and time

5 Pollination Programming

5.1 Mathematical Model of Optimal Profit

In this question, we need to consider where to put our hives at and to organize the number of the bees in each hive. Our goal is to maximize our profit, which means that we need to maximize the number of flowers being fertilized by the bees.[4] To efficiently, address this problem, we assume that the field is a rectangle with 81000 square meters. We therefore, establish a plane rectangular coordinate. Since we define l as the number of the hives, the coordinate of the bee hives are (x_1, y_1), ...,(x_i, y_i), ... , (x_l, y_l).These are all variables.

We assume that the total number of the flowers is N. The coordinate of each of the flower is (n_1, m_1),..., (n_j, m_j), ... , (n_N, m_N). These are all coordinates we already know.

We want to calculate the number of flowers successfully fertilized without getting into very detailed analysis with a single bee's behavior, so we try to calculate the expected value of the number of times which the flowers are being visited by the bees in a single period. We try to analyze this process by considering to aspects.

Firstly, we consider the process of bees choosing flower (see Figure 5.1). Bees in the same bee hives tend to choose different flowers.We assume that bees choose their flowers simply by the distances between the flowers and their beehives. The smaller the distance, the more likely the bees are going to visit this flower. The distance T_{ij} between the ith flower and jth bee hive can be calculated by Euclidean distance:

$$T_{ij} = \sqrt{(x_i - n_j)^2 + (y_i - m_j)^2} \tag{5.1}$$

Secondly, we consider the process of flowers receiving bees (see Figure 5.2). The flowers tend

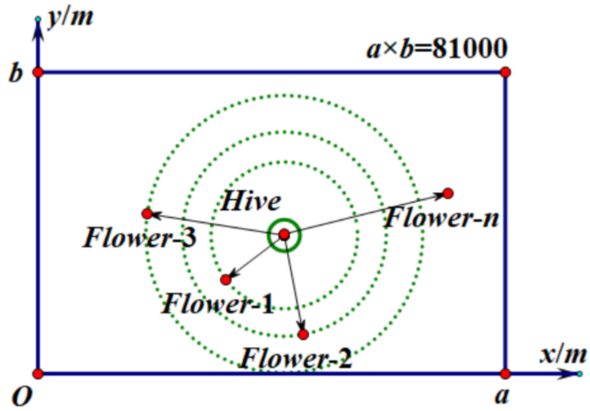

Figure 5.1: Bees in a single hive tend to choose flowers

to welcome all of the bees from different beehives. So we will be able to calculate the whole expected value of the visited number by adding the expected visit times of all the beehives to the flower up.

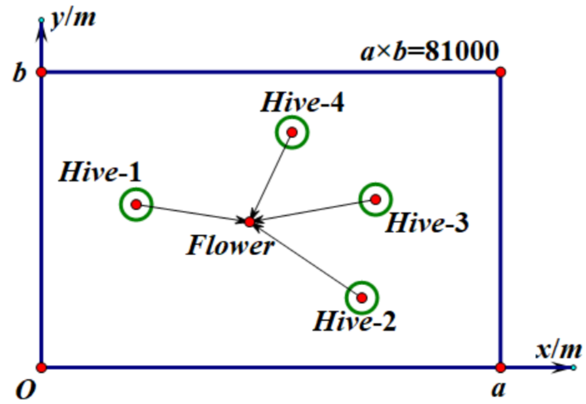

Figure 5.2: Flowers will receive bees from different hives

We calculate the number of bees in the jth bee hive that visit the sth flower. This mainly depends on the ratio of T_{sj} and the total distance from the jth bee hive to all of the flowers,which is $\sum_{i=1}^{N} T_{ij} T_{sj}$ and the number of bees living in the sth bee hive, which is N_j we know that the less T_{sj} is, the more likely will the bees visit from the sth bee hive will visit this flower. We need to add up the variable α and β to represent the working amount as well as the local conditions. We get the following :

$$\left[\frac{\alpha}{(\sum_{i=1}^{N} T_{ij}) T_{sj}} + \beta \right] N_j \tag{5.2}$$

Therefore, we add up all of the bee hives and we can get the expected visited time for flower i:

$$\Delta S_i = \sum_{j=1}^{l} \left[\frac{\alpha}{(\sum_{i=1}^{N} T_{ij}) T_{sj}} + \beta \right] N_j \tag{5.3}$$

In this equation, α and β's value depend on the bee's working ability as well as local factors.

As we all know, some flowers need to be visited several times so that they can grow fruits. We can use this number of times ε as a criteria. If the value of the formula above is more than epsilon. Then we can add 1 to the total number A. By doing this process many times we can get the total A, which is the expected number of flowers that will grow fruits. In this way:

$$A = \sum_{j}^{N} \mathbb{1} \left[\Delta S_i > \varepsilon \right] \tag{5.4}$$

After calculating A, we can try to calculate the benefits of the farmers who manage the beehives.

Firstly, we will calculate the cost of managing the beehives as well as the bees.The cost of installing one bee hive is q, and the number of the bee hives is l. The price of maintaining a single bee is r.

Then we need to calculate what we can get from the fertilized flowers by using the money we can get from one flower if it is fertilized and multiply it with A.The money we can get from a single flower is Z.

Arranging all the possible factors we get the profit formula P. The cost of managing the bee hives: ql; The cost of managing the bees: $r \sum_{i=1}^{l} N_i$; The money we can get from the fruit of the flowers: ZA. So we get:

$$P(x_i, y_i, N_i, l) = ZA - ql - r \sum_{i=1}^{l} N_i \tag{5.5}$$

We only need to find the best coordinate of the beehives x_i,y_i, the number of the beehives l as well as the number of bees N_i living in them to maximize this profit:

$$x_i, y_i, N_i, l = \arg\max P(x_i, y_i, N_i, l) \tag{5.6}$$

5.2 Algorithm Design

The process for solving the problem can be separated into three steps as shown in Figure 5.3. The first step is to decide on variables. We have to decide the number of beehives. Then, to select the location of the beehives, we generate random points for the flower coordinates. We use the density of flowers to create a uniform distribution of points representing individual flowers.

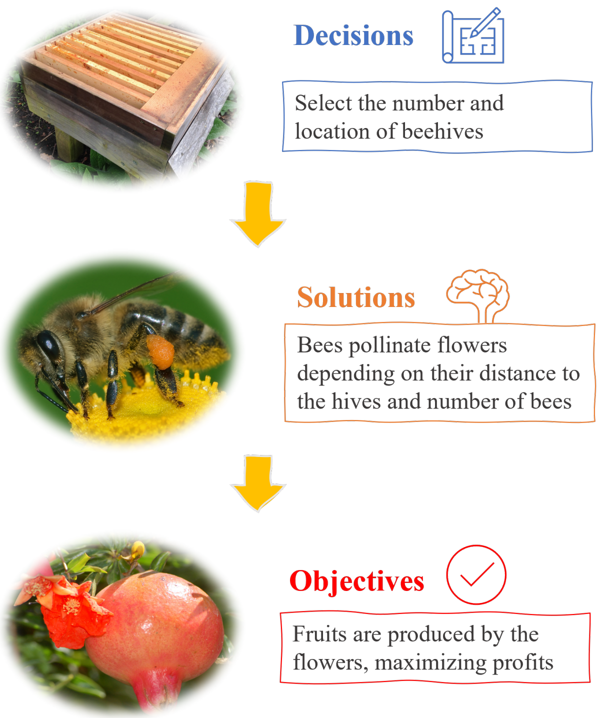

Figure 5.3: Visualization of algorithm

The second step is to find the solution. To find the optimal location of the hive, it is necessary to loop through all possible coordinates. For each location, we would find the distance to every flower T_{sj} and use the formula stated above to calculate ΔS (the number of times visited). The flower is counted as successfully pollinated if the number reaches a threshold value. We can find the optimal location of the beehive by comparing the number of successfully pollinated flowers. We can then calculate the necessary variables and put them into the price function P.

The process described above is only for one beehive. It can be repeated for multiple beehives until the number of iterations reaches some preset value, for which price P can be compared to find the maximum profit possible. The flowchart Figure 5.4 shows this process.

5.3 Numerical Experiment

The model was run for 50 iterations under the following conditions: The cost of caring for one bee in this period(r) equals 0.005, the cost of installing one beehive(q) equals 500, the profit that beekeepers get from when a single flower grows fruit(Z) equals to \$5, and the threshold of pollination(ε) equals to 8.

Figure 5.5 visualizes the distribution of optimal beehive locations. The larger dots represent more beehives, and the smaller ones represent fewer hives in that region. It can be seen that the

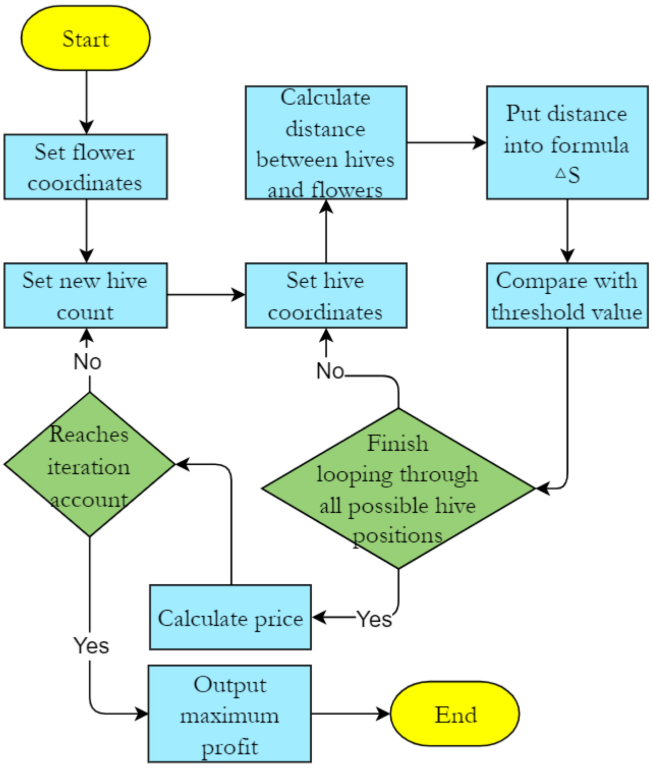

Figure 5.4: Flowchart for algorithm

distribution of the dots is relatively even. However, slightly more hives are collected near the center compared to the corners, with the upper-right corner being the least populated.

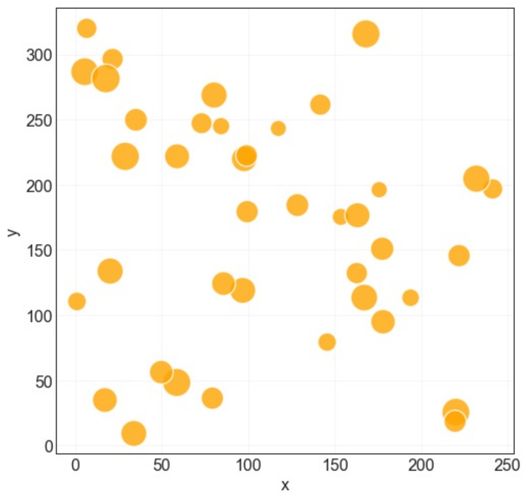

Figure 5.5: Plot of optimal beehive locations

Figure 5.6 shows the relationship between the number of beehives (l) and the total profit (P). Starting from 20 beehives, the profit rises and experiences large fluctuations. For example, there is a dip in profit from around \$23000 to \$17000 when the number of beehives is increased to 25. The graph shows that the profit is maximized at around 37 beehives, bringing in roughly \$31000 in revenue. However, after that spike, the profit decreases continuously, eventually reaching \$10000 at around 48 beehives.

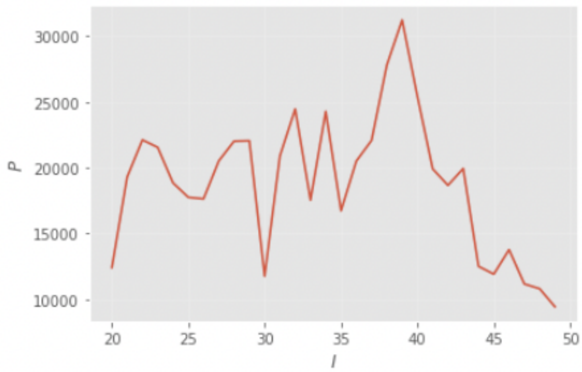

Figure 5.6: Relationship between the number of beehives (l) and the total profit (P)

5.4 Sensitivity Analysis

We performed a sensitivity analysis on the algorithm. The process involved changing one parameter and keeping all other parameters the same. The following table displays the range for which the parameters were changed.

Parameters	Range	Amount generated
r	$0.003 \leq S \leq 0.007$	5
q	$400 \leq m \leq 600$	5
Z	$4 \leq \beta \leq 7$[6]	6
ε	$5 \leq R \leq 12$	8

Table 5.1: Parameter Range

Figure 5.7 shows the relationship between the profit that a single fruit generates (Z) and the total profit (P). The graph shows an upwards trend. The lowest point is Z=4.0, generating less than \$28000, while the highest point is Z=7.0, generating more than \$36000 in profit. This is in line with common sense, as a single fruit generating more profit would mean a group of fruits generating more.

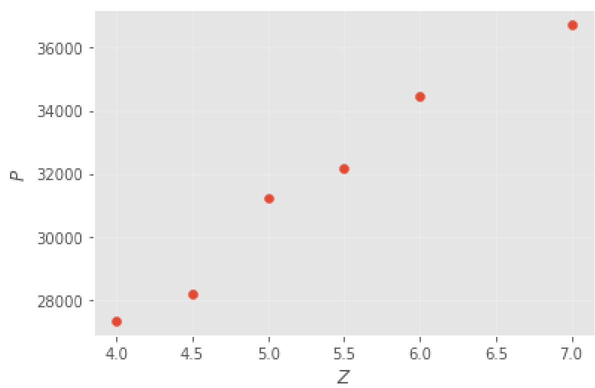

Figure 5.7: Relationship between profits of a single fruit (Z) and total profit (P)

Figure 5.8 shows the relationship between the cost of installing one beehive (q) and the total profit (P). The graph shows a downward trend. The highest point is when q is the smallest, at 400, generating around \$35800 in profit. The lowest point is at 600, generating just above \$22000. This is also in line with common sense, as the increased cost would directly subtract from the profits.

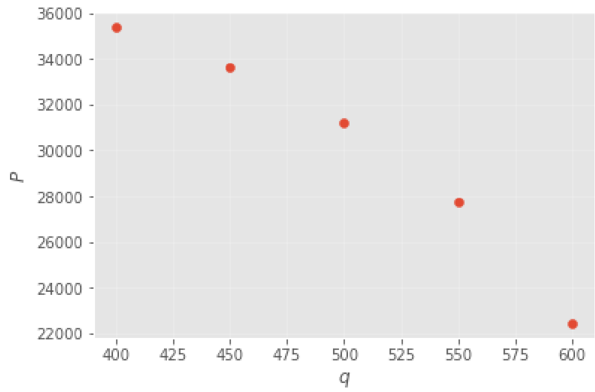

Figure 5.8: Relationship between the cost of installing one beehive (q) and the total profit (P)

Figure 5.9 shows the relationship between the natural death rate of the bee (r) and the total profit (P). The graph shows a downward trend. The highest point is when r is smallest, at 0.0030, generating $34000 in profit. When the death rate is higher, there are fewer bees to pollinate the flowers, so fewer fruits would be grown and less money would be made.

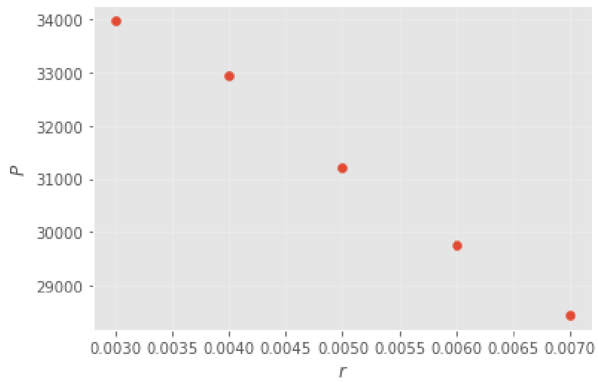

Figure 5.9: Relationship between the natural death rate of the bee (r) and the total profit (P)

Figure 5.10 shows the relationship between the threshold of pollination (ε) and the total profit (P). The graph shows an upwards trend. The lowest point is ε=4.0, generating less than $28000, while the highest point is ε=7.0, generating more than $36000 in profit. This is very similar to the relationship shown in Figure 5.7.

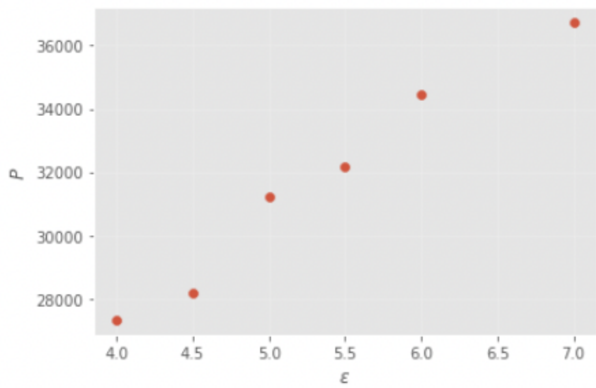

Figure 5.10: Relationship between the threshold of pollination (ε) and the total profit (P)

6 Evaluation

There are both strengths and weaknesses in our model:

Strengths:

- Our model is very comprehensive. In the part calculating the number of hive bees and the foragers. We have taken many factors such as the process of fertilization as well as the predator factors, the virus variables into consideration.
- Our model has practical applications. We have calculated the point of the maximum mortality rate m to warn the bee keepers about the possibility of an extinction.
- Our second model calculating the number of flowers that will successfully grow fruits is concise. We take account of the most important factor in the bees 'flower choices, which is distance between the flower and the bee hives and successfully depict this factor in a scientific way.

Limitations:

- We have not taken bee colony's competition for pollen into consideration in measuring the probability of bees choosing flowers. So our model can be inaccurate when concerning large amount of bees getting pollen in a small area since in this way the competition between bees cannot be neglected.
- We have not consider the impact of the drones on the bee's population. The main role of the drones in a bee colony is to fertilize the eggs. So the fertilizing rate of the eggs is not constant at real life conditions. Therefore, our model may be inaccurate when the population of the drones fluctuate a lot.

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Appendices

Appendix A The Globally Stable Steady State

$$g(H,F) = f(H,F)A$$
$$\frac{dF}{dt} = g(H,F) = mF$$
$$f(H,F) = \frac{H+F}{\omega+H+F}$$
$$\because mF = \frac{(H+F)A}{\omega+H+F}$$
$$\therefore A = \frac{mF(\omega+H+F)}{H+F}$$
$$\frac{dA}{dt} = \beta B - \frac{H+F}{\omega+H+F}A$$
$$\frac{dB}{dt} = (\lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3 + \lambda_4 S_4) * \frac{R-A-B-H}{R} - \beta B - \varphi B$$

To be convenient to our calculation,we regard $\lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3 + \lambda_4 S_4$ as M

$$\beta B = \frac{H+F}{\omega+H+F}A = mF$$
$$\therefore B = \frac{mF}{\beta}$$
$$\frac{M(R-A-B-H)}{R} = \beta B + \varphi B$$
$$\frac{M(R - \frac{mF(\omega+H+F)}{H+F} - \frac{mF}{\beta} - H)}{R} = mF + \frac{\varphi mF}{\beta}$$
$$(\alpha - \sigma\frac{F}{H+F})H = mF$$

Let $\frac{F}{H} = \lambda$,

$$\therefore m\lambda^2 + m\lambda = \alpha\lambda + \alpha - \sigma\lambda$$
$$\therefore \lambda = \frac{-(m+\sigma-\alpha) \pm \sqrt{(m+\sigma-\alpha}^2 + 4m\alpha}{2m}$$

we adopt the root with + , because the other root with - will result λ a negative value, which contradicts our calculation of static value because if one of the H and F is negative and the other is positive, the conversion will definitely happen, which is unacceptable in our static analysis.

$$\therefore \hat{F} = \frac{MR - \frac{Mm\omega}{1+\frac{1}{\lambda}}}{Rm + \frac{R\varphi m}{\beta} + \frac{M}{\lambda} + \frac{Mm}{\beta} + Mm}$$

$$\hat{H} = \frac{MR - \frac{Mm\omega}{1+\frac{1}{\lambda}}}{\lambda(Rm + \frac{R\varphi m}{\beta} + \frac{M}{\lambda} + Mm)}$$

The conditions for $\hat{F}$ and $\hat{H}$ to be larger than 0 is:

$$MR > \frac{Mm\omega}{1+\frac{1}{\lambda}}$$

If $m\omega < R$, Theorem must be valid when $m\omega \geq R$

$$\therefore \lambda < \frac{R}{m\omega - R}$$

$$\frac{-(m+\sigma-\alpha) + \sqrt{(m+\sigma-\alpha^2+4m\alpha}}{2m} < \frac{R}{m\omega - R}$$

$$m^2(\alpha\omega - R) - m(2m\alpha + R\sigma - R\alpha) + \frac{\sigma R^2}{\omega} < 0$$